

3. Atmospheric Thermodynamics

In this section we introduce some of the basic results of thermodynamics and apply them to some simple, but important atmospheric situations.

Perfect Gas Law

In the kinetic theory of gases, an *ideal* gas is one in which the individual molecules are sufficiently far apart that the short range force that acts between them can be ignored. Collisions between molecules are assumed to be perfectly elastic.

Laboratory experiments have shown that for such a gas there is a simple equation of state relating the pressure p , absolute temperature T , and volume V . For m kilograms of gas, this equation may be written

$$pV = mRT, \quad (3.1)$$

where R is a constant for the particular gas. R is called the gas constant and has units of Joules per degree per kilogram.

We define a *kilogram-molecular* weight, or *kilomole* (abbreviated *kmole*) of a material as its molecular weight expressed in kilograms. (Older texts define *molecular weight* as the molecular weight expressed in grams). For example, the molecular weight of water is 18.016, and therefore one kilomole of water is 18.016 kg of water. The number of kilomoles n in mass m (in kilograms) of material is given by

$$n = m/M. \quad (3.2)$$

Where M is the molecular weight. One kilomole of any material is equal to the weight of a single molecule (in kg) times the number of molecules, N . This number is called Avogadro's number and has the value 6.022×10^{26} (for a kmol of substance).

Avogadro hypothesized that gases containing the same number of molecules occupy the same volume at the same pressure and temperature. This implies that for one kilomole of any gas (i.e., $m = M$ from Eq. 3.2)

$$pV = MRT \quad (3.3)$$

from Eq. (3.1). Accordingly, $R^* = MR$ is a universal constant for all gases. It is called the *universal gas constant* and has the value $8314.3 \text{ J deg}^{-1} \text{ kmol}^{-1}$.

For n kmol of any gas, the ideal gas Eq. (3.1) takes the form

$$pV = n R^* T. \quad (3.4)$$

Although air is a mixture of gases with different molecular weights, it is possible to define an apparent molecular weight so that Eq. (3.4) still applies to the mixture.

Consider 1 kg of *dry air* containing m_i kg of the i -th constituent which has molecular weight M_i and exerts a partial pressure p_i . From Dalton's law of partial pressures, the total pressure of the gas mixture, $p = \sum p_i$. Since each component occupies the same volume V , and the number of kilomoles of the i -th component is m_i / M_i , Eq. (3.4) gives

$$p_i V = \frac{m_i}{M_i} R^* T,$$

and summation over i gives

$$pV = \left(\sum_i \frac{m_i}{M_i} \right) R^* T. \quad (3.5)$$

This has the same form as Eq. (3.4) if $n = \sum (m_i / M_i)$. Referring to Eq. (3.2), we see that it follows that the apparent molecular weight of the mixture

$$M_d = \sum_i m_i / \left(\sum_i m_i / M_i \right). \quad (3.6)$$

Note that the apparent molecular weight is not a simple arithmetic average of the constituent species. It makes sense to define an apparent molecular weight for dry air because the proportions of the constituents are very nearly constant.

The *specific volume* of the gas α , is the volume per unit mass (i.e., $V = \alpha$ when $m = 1$). Note that $\alpha = 1/\rho$ where ρ is the density. For dry air, $M_d = 28.97$. Hence, Eq. (3.5) reduces to

$$p\alpha = R_d T, \quad \text{or} \quad p = \rho R_d T \quad (3.7)$$

where

$$R_d = R^* / M_d = 287 \text{ J K}^{-1} \text{ kg}^{-1}.$$

A similar calculation can be carried out for moist air, regarding this as a mixture of dry air and water vapour.

Hydrostatic Balance and Geopotential

The hydrostatic equation

Except for motions on rather small horizontal scales, the atmosphere is in a state of close hydrostatic balance. That is, the net pressure force acting on a small element of air in a vertical column is equal to the weight of air in the element (Fig. 3.1). Mathematically, this force balance can be expressed by the differential relation

$$p(z + \Delta z) A + \rho g A \Delta z = p(z) A,$$

or, as $\Delta z \rightarrow 0$,

$$\frac{dp}{dz} = -g\rho \quad (3.8)$$

This is called the hydrostatic equation. The negative sign arises because pressure decreases with height.

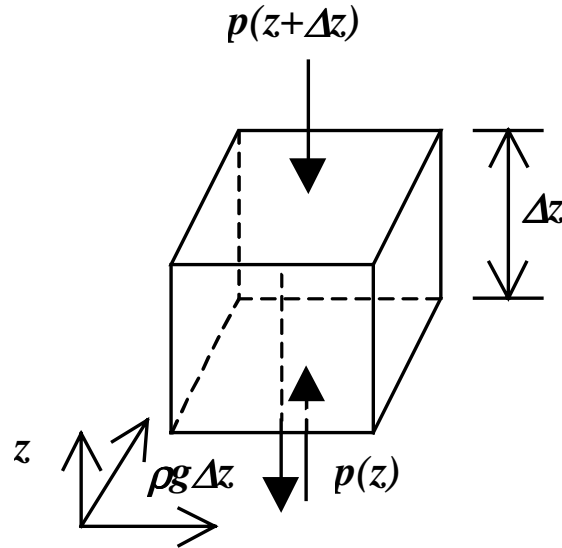


Figure 3.1. Hydrostatic balance.

Given the variation of density with height, $\rho(z)$, Eq. (3.8) may be integrated with respect to z , making use of the conditions that $p(z) \rightarrow 0$ as $z \rightarrow \infty$. Therefore

$$p(z) = \int_z^{\infty} g \rho(z) dz. \quad (3.9)$$

This shows that the pressure at height z is equal to the weight of the air in vertical column of unit area lying above that level.

Alternatively, using the perfect gas equation $p = \rho R_d T$ to eliminate ρ in preference to T , Eq. (3.8) becomes

$$\frac{1}{p} \frac{dp}{dz} = -\frac{g}{R_d T}.$$

This equation integrates at once to give

$$p(z) = p(0) \exp \left[-\frac{g}{R_d} \int_0^z \frac{dz}{T(z)} \right]. \quad (3.10)$$

Equation (3.10) gives the pressure as a function of height in terms of the vertical distribution of temperature $T(z)$ and the surface pressure $p(0)$.

Suppose that the temperature T is constant and equal to \bar{T} in some layer, i.e, the layer is *isothermal*. Alternatively, \bar{T} could be the average temperature in a layer. Then Eq. (3.10) becomes

$$p(z) = p(0) \exp \left(-\frac{gz}{R_d \bar{T}} \right) = p(0) \exp \left(-\frac{z}{H_s} \right), \quad (3.11)$$

where $H_s = R_d \bar{T} / g$ is known as the scale height. The scale height represents an *e*-folding height scale. In other words, the pressure decreases by a factor of 1/e over this height. For a temperature of 288 K near the Earth's surface $H_s \approx 8.5 \text{ km}$. From the perfect gas law, the variation of density with height is given by

$$\rho(z) = \rho(0) \exp \left(-\frac{z}{H_s} \right),$$

where $\rho(0)$ is the density at $z = 0$. Thus, both the pressure and temperature decrease exponentially with height. The atmosphere is compressible under its own weight; lower layers are compressed more than upper layers.

The typical vertical profile of temperature, density and pressure up to a height of about 100 km are shown in Fig. 3.2. The different layers of the atmosphere are defined by the different variations in the temperature with height.

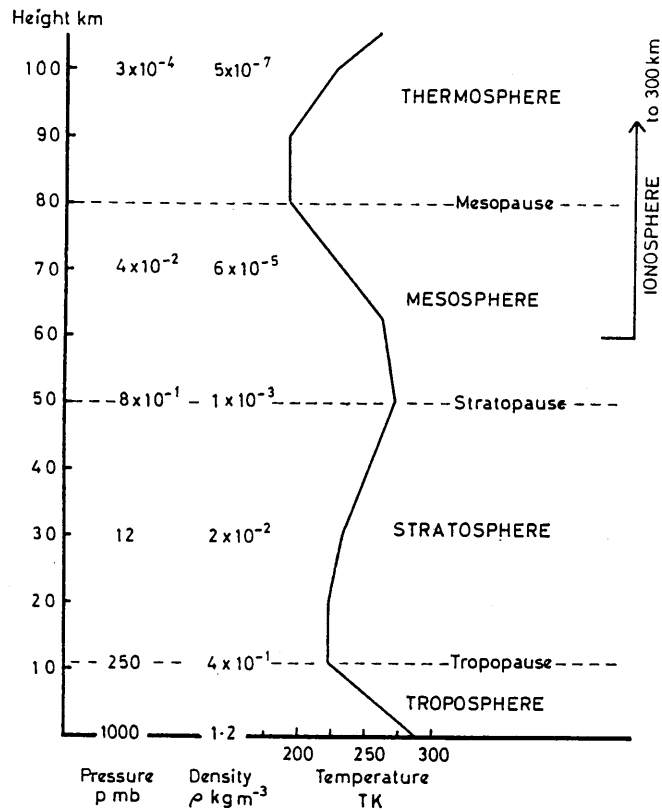


Figure 3.2. Typical vertical profiles of pressure, temperature and density.

Pressure as a vertical coordinate

The use of p as the vertical coordinate is a practical alternative to using z because of the strong hydrostatic relationship that exists in the vertical under many circumstances of interest, and because of the strong functional relationship between pressure and height (i.e., exponential). Figure 3.3 shows how closely height and $\log(\text{pressure})$ are related. It is often easier to measure pressure than the measure height, for example, radiosondes transmit their pressure and temperature from which the height is calculated.

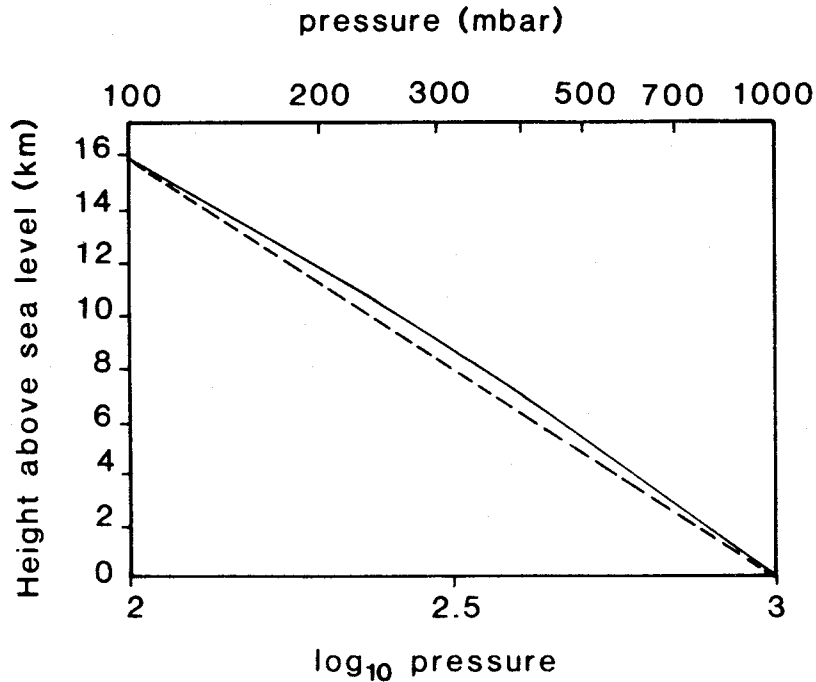


Figure 3.3 Pressure plotted against height for a typical sounding (UK in winter), from McIlveen 1992. The dashed line shows a straight line.

Geopotential

The *geopotential* Φ at any point in the atmosphere is defined as the work that must be done against the Earth's gravitational field in order to raise a mass of 1 kg from sea level to that point. Expressed another way, Φ is the gravitational potential for a unit mass. It has units $J kg^{-1}$, or $m^2 s^{-2}$. The force (in newtons) acting on 1 kg at height z above sea level is numerically equal to g . The work done in raising 1 kg from z to $z + dz$ is $d\Phi = g dz$. Since the depth of the atmosphere is so small in comparison to the radius of the Earth, the acceleration due to gravity is usually taken to be constant ($9.8 ms^{-1}$) in meteorological problems. In this case, $\Phi = g z$. Otherwise, $\Phi(z) = \int_0^z g(z) dz$, assuming $\Phi(0) = 0$.

The *geopotential height* is defined as $Z = \Phi/9.8$. Of course, if g is taken to be a constant, then $z = Z$.

Because of the equation of state,

$$p = \rho R_d T, \quad (3.12)$$

it is necessary to measure only two of the three quantities p , ρ , and T . Since p and T are relatively easy to measure, the density, when required, is obtained from Eq. (3.12).

Wherever possible, it is convenient to eliminate ρ from the equations. For example, using Eq. (3.12) the hydrostatic equation can be written

$$\frac{dp}{dz} = -\frac{p g}{R_d T} . \quad (3.13)$$

Rearranging this, and using the relation $g dz = d\Phi$ gives

$$d\Phi = -R_d T \frac{dp}{p} ,$$

whereupon

$$\Phi_2 - \Phi_1 = -R_d \int_{p_1}^{p_2} T \frac{dp}{p} ,$$

or

$$z_2 - z_1 = \frac{R_d}{g} \int_{p_2}^{p_1} T \frac{dp}{p} . \quad (3.14)$$

Thus, the (geopotential) heights can be determined at a particular place on the Earth by measuring the variation of temperature as a function of pressure. Such measurements are made by radiosonde soundings of the atmosphere.

Upper level synoptic charts in practical use are drawn on isobaric surfaces and the principal isoline on these charts are the geopotential height contours. Since these isobaric surfaces are almost horizontal to a close approximation, the component of air motion in this surface is to within measurable accuracy equal to the horizontal wind speed.

For a dry, isothermal atmosphere $T = \text{constant}$, and Eq. (3.14) may be integrated to give

$$z_2 - z_1 = (RT/g) \ln(p_1/p_2),$$

In the troposphere, the temperature varies with height, but the formula Eq. (3.14) can still be used if we define \bar{T} to be \bar{T} , the mean temperature with respect to $\ln p$, i.e.,

$$\bar{T} = \frac{\int_{\ln p_2}^{\ln p_1} T d(\ln p)}{\int_{\ln p_2}^{\ln p_1} d(\ln p)} = \frac{\int_{p_2}^{p_1} T \frac{dp}{p}}{\ln\left(\frac{p_1}{p_2}\right)} . \quad (3.15)$$

Thickness and height of isobaric surfaces

The difference in (geopotential) height $z_2 - z_1$ between any two levels in the atmosphere is called the *thickness* of the intervening layer. The foregoing theory shows that the thickness between two isobaric surfaces p_1 and p_2 is proportional to the mean temperature between these surfaces. Thus, if \bar{T} increases, the air between the two surfaces expands and the layer becomes thicker. Note that the mass between the two surfaces remains the same.

In weather forecasting offices, charts showing isolines of constant thickness of the layer from 1000 mb to 500 mb are used to depict the regions of cold and warm air in the lower troposphere (see Figure 3.4).

Thickness charts can be readily constructed using data from radiosonde soundings of pressure, temperature and humidity at a network of upper air station.

Reduction of pressure to sea level

The variation of pressure with height is much greater than the horizontal variation across weather systems. This is so much so, that over hilly or mountainous terrain, a map of surface pressure would look like a contour map of the terrain variation. Therefore to isolate the pressure fluctuations associated with the passage of weather systems it is necessary to reduce all pressures to a common reference level, usually *mean sea level*.

One method would be to assume that an isothermal layer of air exists below the station level. Then if the (geopotential) height of the station is z_{st} , the station pressure p_{st} and the station temperature \bar{T}_{st} , the mean sea level pressure $p(0)$ may be estimated by integrating Eq. (3.8); i.e.,

$$p(0) = p_{st} \exp(z_{st} / H), \quad (3.16)$$

where $H = R_d \bar{T}_{st} / g$. If z_{st} is less than a few hundred metres, i.e., $z_{st} / H \ll 1$, a tolerable approximation to Eq. (3.16) is

$$p(0) = p_{st} [1 + (g z_{st} / R_d \bar{T}_{st})].$$

(Recall that $\exp(x) = 1 + x + \dots$ for $x \ll 1$.) For larger station heights, the assumption of a mean isothermal layer below the station becomes questionable and empirical corrections are applied. Over mountainous terrain, the reduction to sea level pressure is not entirely satisfactory.

Figure 3.4 shows the mean sea level pressure (MSLP) and the 1000-500 mb thickness for a typical summertime situation in Australia. The main features include the surface cold front over southeastern Australia sandwiched between two anticyclones. The combination of MSLP and 1000-500mb thickness patterns show hot continental northerlies precede the front and cooler maritime southwesterlies follow it.

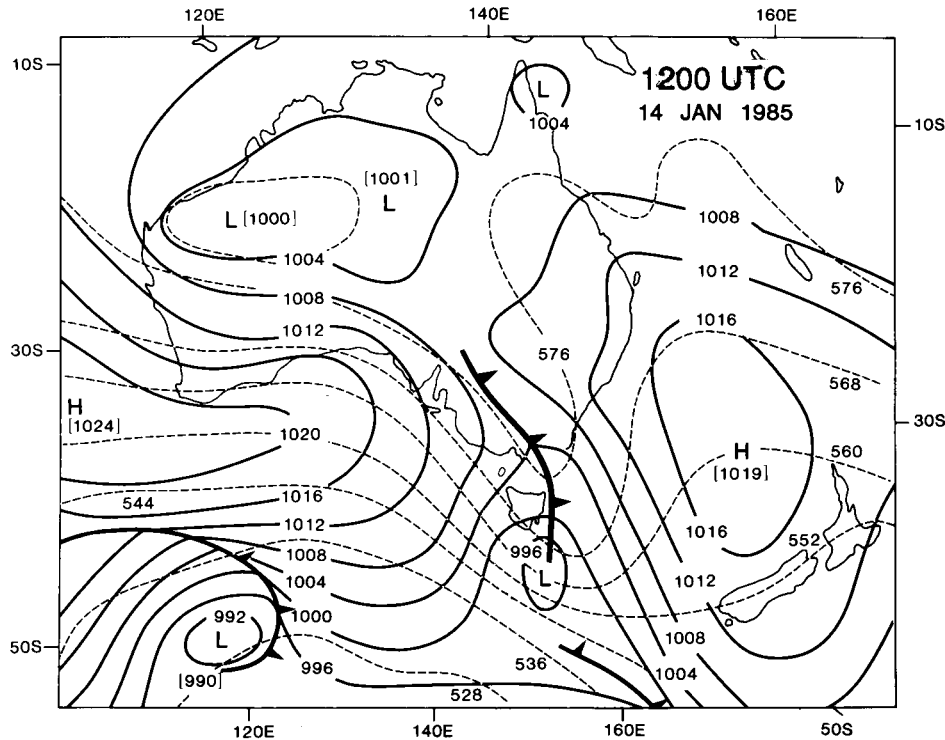


Figure 3.4. Mean sea level pressure (MSLP) (solid lines) and the 1000-500 mb thickness (dashed lines). The labels on the thickness contours are in decametres (10 m intervals), and hence the contour interval is 80 m.

Potential Temperature

The stability of the atmosphere may seem counter-intuitive. We are aware from our everyday experience that hot air rises, so why is it that the troposphere is stable despite the temperature decreasing with height? In the following sections we shall investigate why this is so. It will be shown that the stability of the troposphere depends on both the temperature and the decreasing pressure with height. The pressure and temperature can

be combined into a single variable, the potential temperature, and it will be shown that the atmosphere is stable if the potential temperature increases with height. We shall define the potential temperature, θ , to be a quantity that will not change as air parcels are moved adiabatically (without heat exchange).

The first law of thermodynamics

Suppose that a small quantity of heat, dq , is added to a unit mass of gas. Conservation of energy requires that the heat added be balanced by an increase in the internal energy of the gas, du , and work done by the gas against its environment, dw . This may be expressed mathematically as

$$dq = du + dw, \quad (3.17)$$

and is known also as the *first law of thermodynamics*.

For a unit mass of gas, the volume V is simply the specific volume α . Then the work done *by* the gas when its specific volume increases by $d\alpha$ is

$$dw = p d\alpha.$$

Note that the work done = force \times distance. For example, for a unit mass of gas with cross-sectional area A , $d\alpha = A dx$, or $p d\alpha = (p A) dx = \text{force} \times \text{distance}$. (See Fig. 3.5.)

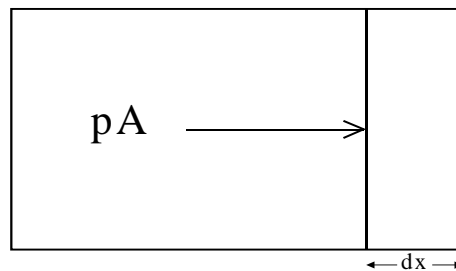


Figure 3.5. Work done in displacing a boundary of area A a distance dx .

Hence Eq. (3.17) becomes

$$dq = du + p d\alpha. \quad (3.18)$$

Suppose that the heat added to the gas increases its temperature from T to $T+dT$ without a change in phase. The ratio dq/dT is called the *specific heat*. The specific heat defined in this way can have any number of values depending on how the gas changes as the heat is added.

For example, if the volume of the gas remains constant, the ratio dq/dT , denoted by c_v , is called *the specific heat at constant volume*. In this situation, no work is done by the gas in expanding, whereupon Eq. (3.18) gives $dq = du$, i.e.,

$$c_v = \left(\frac{du}{dT} \right)_{\alpha=\text{constant}} . \quad (3.19)$$

However, the internal energy of an ideal gas is independent of volume since the molecules of an ideal gas do not exert any attractive or repulsive forces on each other. (In fact, it can be shown that the average kinetic energy of a gas molecule = $3kT/2$, where $k = R^*/N$ is Boltzman's constant.) Consequently, the internal energy depends only upon the temperature, in which case Eq. (3.19) gives

$$du = c_v dT.$$

Hence, Eq. (3.18) can be re-expressed as

$$dq = c_v dT + p d\alpha. \quad (3.20)$$

As a second example, consider a case in which the heat is added to the gas in such a way that the pressure remains constant. Then we may define a *specific heat at constant pressure* as

$$c_p = \left(\frac{dq}{dT} \right)_{p=\text{constant}} \quad (3.21)$$

In this case, some of the heat input dq will go into doing work as the gas expands and pushes against its environment. Therefore, a larger quantity of heat will need to be added to the gas to raise its temperature by the same dT as in the case $\alpha = \text{constant}$.

To show this we write the perfect gas equation in the differential form (using the product rule)

$$pd\alpha + \alpha dp = R_d dT. \quad (3.22)$$

Using Eq. (3.22) to eliminate $pd\alpha$ from Eq. (3.20) gives

$$dq = (c_v + R_d) dT - \alpha dp. \quad (3.23)$$

At constant pressure, $dp = 0$, whereupon Eq. (3.21) gives

$$c_p = c_v + R_d. \quad (3.24)$$

For dry air, $c_v = 717 \text{ J deg}^{-1} \text{ kg}^{-1}$.

Combining Eqs. (3.23) and (3.24) we obtain an alternative form of the first law of thermodynamics, i.e.,

$$dq = c_p dT - \alpha dp. \quad (3.25)$$

Note that αdp is *not* the work done, but for an isothermal process $p d\alpha + \alpha dp = 0$, and hence $dw = p d\alpha = -\alpha dp$. (An isothermal process is one for which the temperature remains constant, and hence $dT = 0$.)

Adiabatic changes

As an air parcel moves around in the atmosphere, it tends to receive little external heat input, at least on a time scale of a few hours. This is due to the relatively low thermal and radiative conductivity of air. Thus, air parcels carry their heat with them and suffer negligible heat loss or gain. Accordingly, it is a good approximation to assume that changes of state are adiabatic.

The mathematical statement that no heat is gained or lost is $dq = 0$. Thus, from Eq. (3.20)

$$0 = c_v dT + p d\alpha.$$

If the air parcel expands ($d\alpha > 0$), it must cool ($dT < 0$) as it does work ($p d\alpha$) against its environment.

Alternatively, from Eq. (3.25),

$$0 = c_p dT - \alpha dp.$$

Using the perfect gas equation, this becomes

$$0 = c_p dT - \frac{R_d T}{p} dp$$

or, upon dividing by T ,

$$\frac{dT}{T} - \kappa \frac{dp}{p} = 0,$$

where $\kappa = R/c_p$. Therefore, integrating from (T, p) to (θ, p_*) gives

$$\ln T - \kappa \ln p = \ln \theta - \kappa \ln p_*, \quad (3.26)$$

where θ and p_* are reference values of temperature and pressure. For *dry air*, $\kappa = 0.286$. Alternatively, Eq. (3.26) may be written as

$$T = \theta \left(\frac{p}{p_*} \right)^\kappa . \quad (3.27)$$

θ is called the *potential temperature* of the air parcel. Physically, the potential temperature of an air parcel is the temperature the parcel would have if it were brought adiabatically to the standard pressure p_* ; generally we take $p_* = 1000$ mb. Since the potential temperature is just the constant of integration, it is a conserved quantity for an air parcel in *adiabatic motion*, i.e., motion in which there are no heat sources or sinks.

In meteorology, it proves extremely useful to label air parcels with their potential temperature. In fact, the potential temperature is defined by

$$\theta = T \left(\frac{p_*}{p} \right)^\kappa . \quad (3.28)$$

We shall see that the vertical stability of a dry atmosphere can be characterized by the variation of θ with height. Meteorologists have developed *aerological diagrams* which allow the pressure and temperature to be plotted and the potential temperature to be read off.

Aerological Diagrams

There are many different types of aerological diagrams, but they all share some important characteristics. All of them allow points to be plotted according to their temperature and pressure, and the potential temperature to be read off. In addition, information about moist processes, such as the likelihood of cloud formation, can be obtained from the diagrams. However, in this course, we only consider dry processes.

The aerological diagram used in this course is the skewT-logP diagram used by the Australian Bureau of Meteorology. As the name suggests, the vertical axis is log(pressure) and the other axis is skew (diagonal) temperature. A simplified version of this diagram is shown in Figure 3.6. The horizontal lines are lines of equal pressure and are labelled in hPa, the dashed lines (positive slope) are lines of constant temperature and are labelled in $^{\circ}\text{C}$. The negatively sloping solid lines are *dry adiabats*, and indicate lines of equal potential temperature. Another way to think about dry adiabats is to realise that motion along these lines is adiabatic motion – no heat is exchanged.

Remembering the definition of potential temperature, it is easy to find the potential temperature given the pressure and temperature. Plot the (pressure, temperature) reading on the aerological diagram, then follow a dry adiabat to $p=1000$ hPa and read off the temperature at this point.

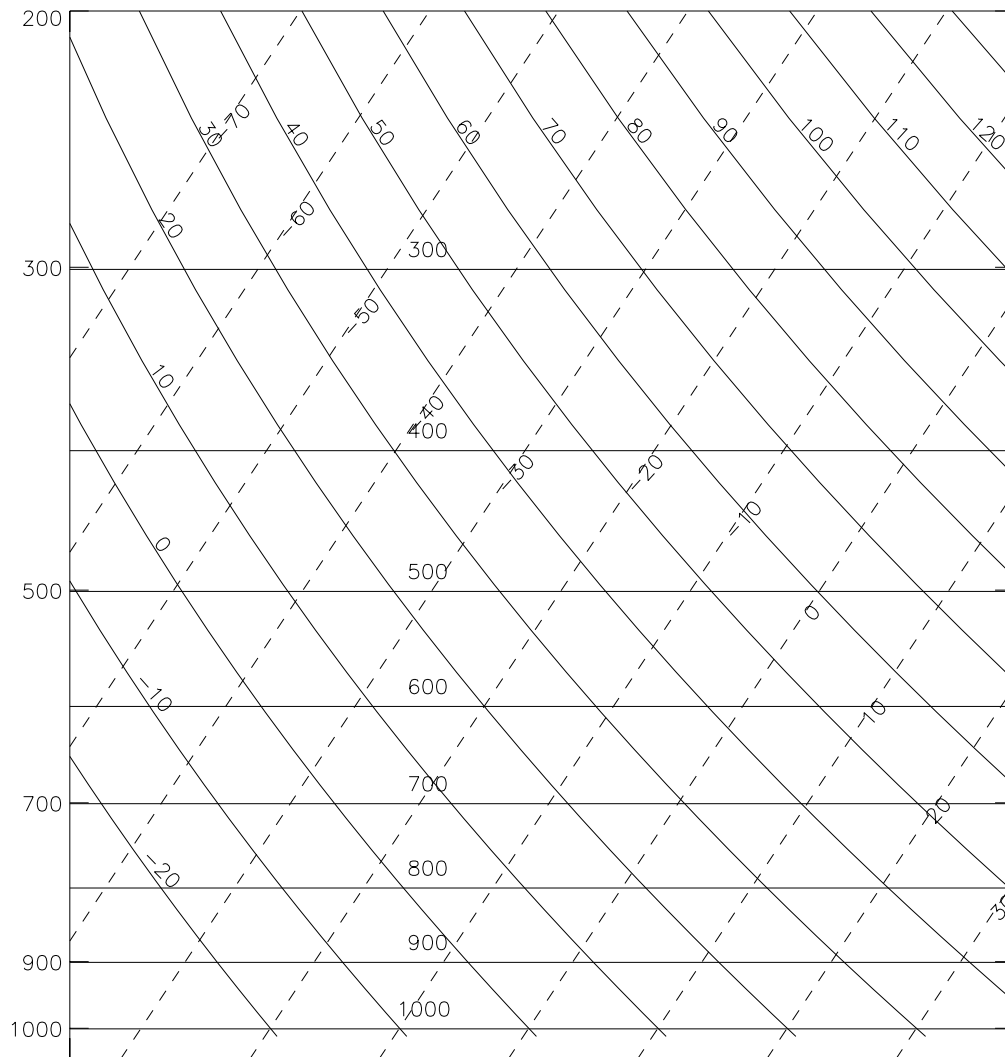


Figure 3.6 Simplified aerological diagram.

Equilibrium and stability

Assume that the temperature varies with height, and suppose that a parcel with volume V is lifted a small distance dz from its equilibrium position at point A to a neighbouring point B (see Fig. 3.7). For $dz > 0$, the gas pressure at $z+dz$ is less than that at height z . Thus, as the particle rises, it expands and cools. In contrast, if it is displaced downwards, it is compressed and heated.

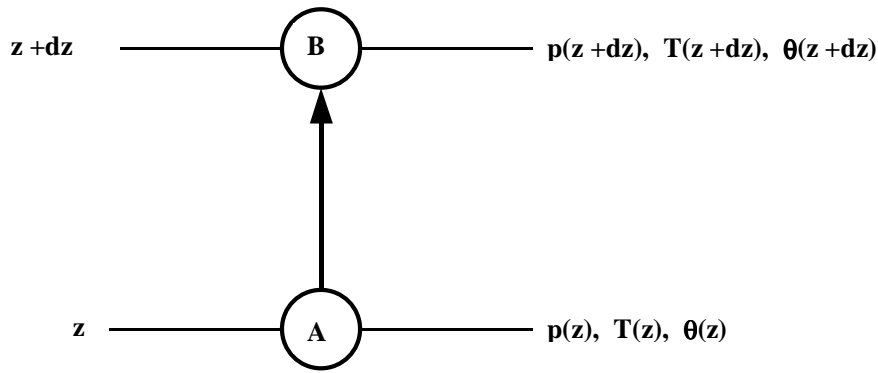


Figure 3.7. An air parcel lifted from its equilibrium position.

Although the displaced parcel's temperature and pressure change, its potential temperature remains constant and equal to its original value at A . Since the pressure at level B is $p(z + dz)$, the temperature of the parcel after displacement from A to B , given by the conservation of potential temperature,

$$T_B = \theta(z) [p(z + dz)/p_*]^\kappa. \quad (3.29)$$

Again using the definition of potential temperature, the temperature of the parcel's environment at level B is

$$T(z + dz) = \theta(z + dz) [p(z + dz)/p_*]^\kappa. \quad (3.30)$$

In general, the displaced parcel will experience a nonzero force at its new position. According to Archimedes principle, the force per unit mass, F , experienced by the parcel at B is

$$\begin{aligned} F &= \frac{(\text{weight of air displaced} - \text{weight of air in parcel})}{\text{mass of air in parcel}} \\ &= \frac{(g\rho(z + dz)V - g\rho_B V)}{\rho_B V}, \end{aligned}$$

where ρ_B is the parcel's density at level B . F , is called the *buoyancy force per unit mass*. Cancelling V and using the perfect gas law $\rho = p(z + dz)/RT$, the above expression gives

$$F = g(T_B - T(z + dz)) / T(z + dz).$$

Using Eqs. (3.29) and (3.30), this becomes

$$F = g(\theta(z) - \theta(z + dz)) / \theta(z + dz).$$

Now F is expressed entirely in terms of θ which is a measurable property of the environment, and can be written approximately as

$$F \approx -\frac{g}{\theta} \frac{d\theta}{dz} dz = -N^2 dz. \quad (3.31)$$

If the buoyancy force (per unit mass) is directed from B to A (i.e., in the opposite direction to the parcel's displacement), it will push the parcel back towards point A. In this case the atmosphere is *stable* to small parcel displacements. Conversely, if F is directed from A to B, the parcel will accelerate away from point A, and therefore the atmosphere is *unstable* to small parcel displacements. Thus, the criterion for stability in a compressible atmosphere depends on the sign of the vertical gradient of potential temperature.

Returning to the displaced parcel and the case of stable stratification, assume that the subsequent particle motion is caused only by the buoyancy force and that the effect of the surrounding fluid can be neglected. We may then write down Newton's second Law for the parcel, i.e.,

$$\text{acceleration} = \text{force per unit mass},$$

$$\text{i.e.,} \quad \frac{d^2\xi}{dt^2} + N^2 \xi = 0, \quad (3.32)$$

where

$$N^2 = \frac{g}{\theta} \frac{d\theta}{dz}, \quad (3.33)$$

and $\xi(t)$ now denotes the displacement dz .

If the potential temperature increases with increasing z , then N^2 is a positive constant and Eq. (3.32) is the simple harmonic equation. In this case, it has a solution of form $\xi = \xi_0 \cos(Nt + \chi)$, where ξ_0 and χ are constants. Thus, the air parcel oscillates with frequency N called the Brunt-Väisälä frequency or simply the buoyancy frequency. The corresponding period, $2\pi/N$, is called the Brunt-Väisälä period. On the other hand, if the potential temperature decreases with height, N^2 is negative and Eq. (3.32) has an exponentially-growing solution proportional to $\exp(Nt)$. Thus, when $N^2 > 0$, the atmosphere is stable to small displacements, whereas the atmosphere is unstable to small displacements when $N^2 < 0$.

In general, N is a function of height. Nonetheless, the character of the solution to Eq. (3.32) is still oscillatory if $N^2 > 0$ and has exponential behaviour if $N^2 < 0$.

It is not surprising that N turns out to be a key parameter in the theory of gravity waves in the atmosphere, for it characterizes degree of gravitational stability

In general, for our stratified fluid calculations we assumed that the parcel motion was adiabatic, i.e., that heat diffusion into the parcel from its surrounding is "slow". We neglected heating or cooling on the time scale of a parcel to oscillate one period, i.e., $2\pi / N$. In the troposphere, this period is typically 10 minutes.

Except possibly close to the ground on a sunny day, unstable layers are not observed in the atmosphere. Even a slight degree of instability becomes neutrally stable, i.e., $d\theta / dz = 0$. We refer to this mixed-up state as one of neutral stability.

During the day, when the ground is heated by solar radiation, the air layers adjacent to the ground are constantly being overturned by thermal convection to give a neutrally-stable well-mixed layer with a uniform potential temperature. In dry desert regions in the subtropics, the convective layer may be up to 4 km deep. In Melbourne on a warm summers day it is on the order of 1-2 km, depending on the surface moisture. If the ground surface is very moist, much of the sun's heat goes into evaporating water vapour, reducing the amount available to heat the air layer near the ground. At night, if the wind is not too strong, and especially if the air is dry and there is a clear sky, a strong *radiation inversion* forms in the lowest layers. An *inversion* is a layer of air in which not only the potential temperature, but also the temperature increases with height; such a layer is very stable. Typically, nocturnal radiation inversions are only a few tens of metres deep.

Vertical profiles of potential temperature

Figure 3.8 shows vertical profiles of potential temperature for a sequence of radiosonde soundings. The soundings span the passage of a cold front though Mt Isa. The front arrived at Mt Isa at 0940 EST 10 September 1991. The soundings at 2100 EST 9 September and at 0600 and 0900 EST 10 September are therefore about 13, 3 and 1 hours before the arrival of the front. The soundings at 1200 and 1500 EST are about 2 and 5 hours after the passage of the front respectively.

Only the lowest 5 km is shown. The unstable layer (negative $d\theta / dz$) near the ground during the day caused by strong insolation. The layer of almost constant θ above this layer is evidence that convective mixing is taking place. Above the neutral layer the profile is stable. By evening (2100) the atmosphere has cooled markedly in the lowest 100-200 m. The sounding at 1500, almost five hours after the passage of the cold front, there has been a very marked reduction in temperatures below 1.5 km.

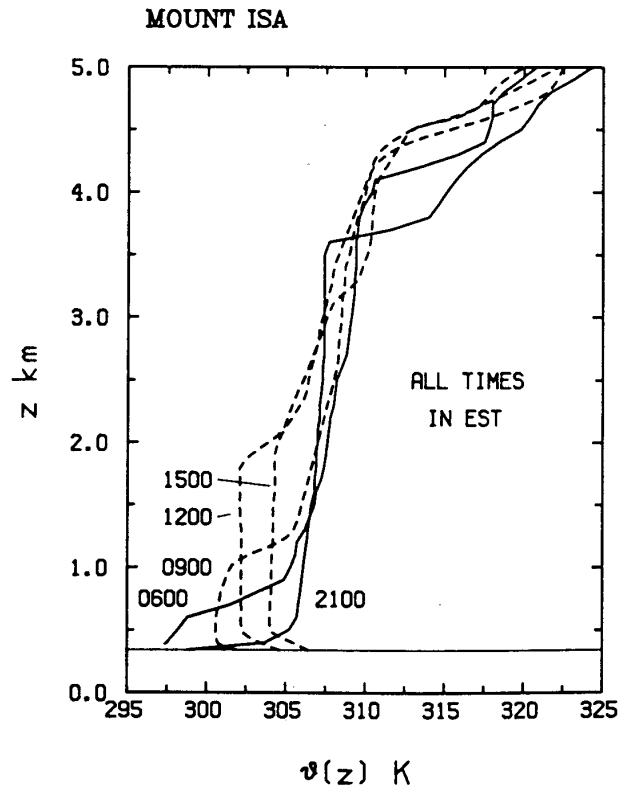


Figure 3.8. Vertical profile of θ for five radiosonde soundings made at Mt Isa.

Lapse rates

We saw earlier that on average, the temperature in the troposphere decreases with height. The magnitude of the rate of decrease is called the *temperature lapse rate*, or just the *lapse rate*. It is sometimes denoted by $\Gamma (= -dT / dz)$.

Consider now the lapse rate in an atmosphere in which the potential temperature is constant. From the definition of θ in Eq. (3.28), we have

$$\ln T = \ln \theta + \kappa (\ln p - \ln p_*).$$

Differentiating with respect to z gives

$$\frac{1}{T} \frac{dT}{dz} = \frac{1}{\theta} \frac{d\theta}{dz} + \frac{\kappa}{p} \frac{dp}{dz}.$$

Using the hydrostatic relation ($dp/dz = -g\rho$), the perfect gas law, and remembering that $\kappa = R / c_p$, gives

$$\frac{dT}{dz} = \frac{T}{\theta} \frac{d\theta}{dz} - \frac{g}{c_p}.$$

However, if θ is constant with height and if the air layer in question is in hydrostatic equilibrium, $d\theta/dz = 0$, and hence

$$\frac{dT}{dz} = -\frac{g}{c_p}, \quad (3.34)$$

This gives the rate at which temperature falls with height in a layer of dry air that has uniform potential temperature; for example the convectively well-mixed layer discussed above. It is very nearly 10°C per kilometre.

The same calculation applies to the change of temperature of an air parcel rising dry adiabatically. Accordingly, the temperature of such a parcel decreases with height at the dry adiabatic lapse rate.

Concept of an air parcel

From time to time in our discussion so far we have talked about an "air parcel" and it is appropriate that we clarify this concept. In many fluid mechanics problems, diffusion is important only within a centimetre of the earth's surface and at levels above the turbopause (~105 km). At intermediate levels virtually all the vertical mixing is accomplished by the exchange of well-defined air parcels with horizontal dimensions ranging from a few centimetres to the scale of the earth itself.

We have sought to gain insights into the nature of vertical mixing in the atmosphere by considering the behaviour of an air parcel of infinitesimal dimensions that is assumed to be:

- thermally insulated from its environment so that its temperature changes adiabatically as it rises or sinks,
- always at exactly the same pressure as the environmental air at the same level, which is assumed to be in hydrostatic equilibrium, and
- moving slowly enough that its kinetic energy is a negligible fraction of its total energy.

In the case of real air parcels one or more of these assumptions is nearly always violated to some extent. However, this simple idealized model is helpful in understanding some of the physical processes that influence the distribution of vertical motion and vertical mixing in the atmosphere.