4. Dynamical Meteorology

Forces felt by an air parcel

The atmosphere is a fluid and is, therefore, subject to the laws of fluid motion. Central among these laws is Newton's second law of motion which states that

$$mass \ ´ \ acceleration = force. \tag{4.1}$$

However, in contrast to the case of rigid-body dynamics, we need to apply an additional constraint, a continuity equation, or mass-conservation equation. Because of this constraint, it is not possible, in general, to specify the "force", or more precisely "force field" independently, as in rigid body problems. An additional complication is that the Earth rotates about its axis with an angular velocity Ω equal to 2π radians per day, or 7.3×10^{-5} radians per second. Therefore, a coordinate frame fixed to the Earth does not provide an inertial frame of reference in which to calculate the "acceleration" in Eq. (4.1).

Suppose that an air parcel with unit volume has acceleration $\underline{a}$ in an inertial frame and $\underline{a}'$ in a frame of reference moving with the Earth. Let $\underline{a}'' = \underline{a} - \underline{a}'$. Then Eq. (4.1) can be written in either of the forms

$$r\,\underline{a} = r\,\underline{a}' + r\,\underline{a}'' = \underline{F} \tag{4.2a}$$

or

$$r\,\underline{a}' = \underline{F} - r\,\underline{a}'', \tag{4.2b}$$

where $\underline{F}$ is the force acting on the air parcel. Equation (4.2a) shows that when we calculate the acceleration relative to the rotating frame, we must add an extra acceleration term, $r\,\underline{a}''$, on the left hand side of Newton's equation to "correct for" the acceleration of the frame. Alternatively, we can put this term on the right-hand side of the equation as in Eq. (4.2b) whereupon we interpret it as a "force". This is commonly referred to as an "apparent" or "fictitious" force, an unfortunate description, because to an observer in the moving frame, it is just as "real" as any other force. In fact, it is generally most convenient to represent atmospheric motions in a frame of reference fixed on the Earth. We shall express the acceleration in this frame and regard the additional terms that arise in doing so as "forces". First we consider how to represent the force field.

In writing down a mathematical expression for Eq. (4.2b), it is usual to consider a small air parcel with a regular shape such as a rectangular box with sides parallel to three rectangular coordinate axes (x, y, z) as in Fig. 4.1. It is conventional to choose z pointing vertically upwards. If the air parcel has dimensions $\Delta x, \Delta y, \Delta z$ in the three coordinate directions, its volume is $\Delta x \Delta y \Delta z$ and its mass is $\rho \Delta x \Delta y \Delta z$ where ρ is the average density of the parcel (remember density = mass per unit volume).

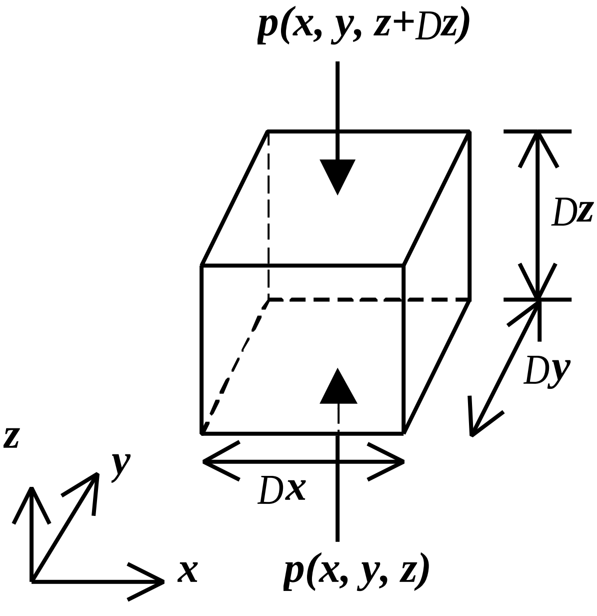


Figure 4.1. The balance of forces on a rectangular fluid element.

There are three kinds of force acting on the air parcel: *body forces*, *pressure forces* and *frictional stresses*. Body forces are forces that are proportional to the mass of the air, they include the gravitational force, just the weight of the fluid which acts downwards and, in the rotating frame, contributions from $r\,\underline{a}''$.

Pressure forces act normal to the sides of the air parcel. Pressure is a *force per unit area*. If the pressure is uniform over each horizontal surface of the box shown in Fig. 1, the *net* pressure force acting in the positive *z*-direction is

$$p(x,y,z)\,\Delta x\,\Delta y - p(x,y,z+\Delta z)\,\Delta x\,\Delta y.$$

For small Δz, the net pressure force per unit volume acting in the z-direction is approximately

$$\frac{-(\,p(x,y,z+\Delta z) - p(x,y,z))}{\Delta z}.$$

Taking $\Delta z \rightarrow 0$ gives $-\P\, p/\P z$ which is the *pressure force per unit volume*, and $-(1/r)(\P\, p/\P z)$ is the *pressure force per unit mass*.

For a body of air at rest, the net upward pressure force exactly balances the downward gravitational force, $-g\,r\,\Delta x\,\Delta y\,\Delta z$, which leads to the hydrostatic equation derived earlier, *i.e.,*

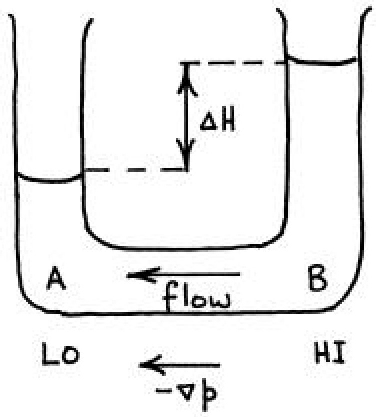
$$\frac{\P p}{\P z} = -r\,g. \qquad (4.3)$$

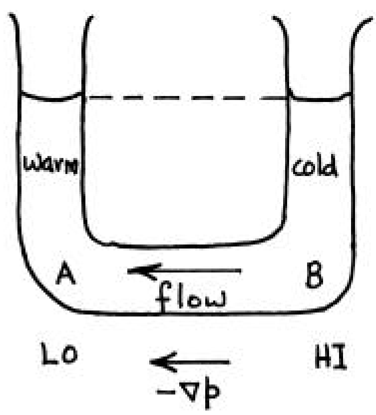
In general, when the air is in motion, there will be components of the pressure force per unit volume, or pressure gradient, in the x and y directions, equal respectively to $-\left(\partial p/\partial x\right)$ and $-\left(\partial p/\partial y\right)$. Thus, the total pressure force per unit volume acting on the air parcel is the *vector* $-\left(\partial p/\partial x\right)\underline{i}-\left(\partial p/\partial y\right)\underline{j}-\left(\partial p/\partial z\right)\underline{k}$. Note that this vector is sometimes given the symbol $-\nabla p$. Likewise, the total pressure force per unit mass acting on the air parcel is $-(1/\rho)\nabla p$.

In contrast to pressure forces, *frictional forces* have components parallel to, as well as normal to, the sides of the box and the derivation of an expression representing their effect is quite complicated. Fortunately, under many circumstances, they can be neglected to a good first approximation in comparison with other forces and for the present we shall not consider them in detail.

In large and medium scale atmospheric motions, it turns out that the hydrostatic equation (4.3) remains satisfied to a high degree of approximation. In other words, the vertical accelerations can be totally ignored in comparison with the gravitational acceleration, g. This is true for low and high pressure systems, cold and warm fronts and even tropical cyclones. It is not true in convective clouds or thunderstorms where significant vertical accelerations can occur in strong updraughts and downdraughts. However, for the larger scale motions, it is essentially the horizontal component of the pressure gradient force that is responsible for producing winds. Horizontal pressure gradients may arise for a variety of reasons. Two important examples are differences in fluid depth, or horizontal temperature differences as depicted schematically in Fig. 4.2.

Horizontal pressure gradients in the atmosphere can be measured easily using barometers, at least if the terrain is level, *e.g.,* between two stations at mean sea level. A typical pressure difference between the centre of a low pressure system, or extra-tropical depression and its environment is 10 mb over a distance of 1000 km. The corresponding pressure gradient per unit mass of air is 10^3Pa/10^6m $\div\ \rho$ which is approximately 10^{-3}ms^{-2} since at the surface the density of air is on the order of 1 kg/m^3. A pressure gradient of this magnitude acting alone on a unit mass of air for 1 day (~ 10^5 sec) would lead to an increase in its velocity by 100 ms^{-1}, typically an order of magnitude larger than is normally observed. We shall see that for large-scale systems such as this, a greater part of the horizontal pressure gradient force is balanced by an inertial force associated with the Earth's rotation, included in the term $\rho\underline{a}''$ on the right-hand side of Eq. (4.2b). Before we derive an expression for $\underline{a}''$, we shall discuss a thought-experiment to illustrate the principal idea.





(a) homogeneous fluid (b) inhomogeneous fluid

Figure 4.2. Two examples of flow caused by horizontal pressure gradients. In (a), the fluid is assumed to be homogeneous (i.e., of constant density). The pressure at B is higher than that at A by an amount equal to the weight of the fluid in the shaded region (of depth ΔH) above B, assuming of course that the pressure at the free surface above B is the same as that at A. Accordingly, the flow is from B to A, i.e., "down" the pressure gradient, from high to low pressure. In (b) the fluid level above A is the same as that above B, but the density is lower because the fluid is warmer. Again this means that the pressure at B is higher than at A because the weight of fluid is greater and again the flow is "down" the pressure gradient.

Motion on a merry-go-round

Imagine that you are the centre of a merry-go-round that is rotating *anticlockwise* with a uniform angular velocity Ω (Fig. 4.3). You roll a ball to a friend who is standing outside at the edge of the merry-go-round. Neglecting the effect of friction and of the Earth's rotation (even your friend is in a rotating frame, the Earth, but the Earth's angular rotation is negligible in comparison with that of the merry-go-round), your friend will observe that the ball travels in a straight line with uniform speed, the speed with which it left your hand, and will conclude that no force acts on it in accordance with Newton's second law. You will observe that the ball curves to the right and will conclude that it must be acted upon by a force, again in accordance with Newton's second law. Who is correct? Of course, you both are. Your friend observes the acceleration, or actually the absence of any acceleration, in an inertial frame (neglecting the Earth's rotation) and applies Newton's law in the form Eq. (4.2a) and concludes that $\underline{F} = \underline{0}$. You observe an acceleration $\underline{a}'$ in the rotating frame and conclude that there *is* a force. The force you observe, *i.e.*, the one that produces the acceleration $\underline{a}'$ that you measure, is simply $-\underline{a}''$. This force is called a *Coriolis force*. It is the force required to be added to the right-hand side of Newton's second law when you measure the acceleration in a rotating frame.

You may be worried by the foregoing arguments because you will be aware that the merry-go-round is fixed on the Earth which is itself rotating. The arguments are valid provided the angular velocity of the merry-go-round is large compared with that of the Earth (= 1 revolution or 2π radians per day).

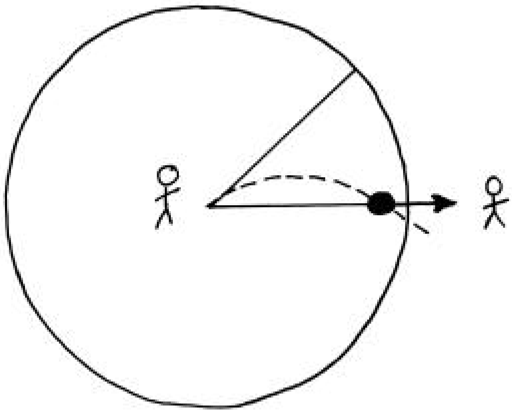


Figure 4.3. A thought experiment illustrating the Coriolis force.

If the angular velocity Ω of the merry-go-round is increased while the speed at which you roll the ball remains the same, you will observe an increase in the curvature of the ball's trajectory and will conclude that the Coriolis force increases with Ω. However, beware! We cannot deduce from the same type of argument that as the speed of the ball V increases the Coriolis force decreases; in fact, the Coriolis force increases. The curvature argument then breaks down because when the ball is travelling more quickly, the time available for the Coriolis force to act over a given distance is reduced. In fact, the Coriolis force is *directly proportional* to both Ω and V. We can do a simple calculation to show this.

Consider Fig. 4.4. Let OA' be a line fixed in space with O at the centre of the turntable and A' at the edge, at radius r. Let OA be a similar radial line moving with the turntable. The time t taken for the ball to reach A' from O is simply r/V. During this time A has moved a tangential distance $s = (\Omega t) \times r = \Omega V t^2$. Differentiating this expression twice with respect to t gives $2\Omega V$. This is the acceleration seen by the observer at the centre of the turntable. Thus, the observer would conclude that the ball is subject to a force per unit mass $2\Omega V$ to the right, *i.e.,* a Coriolis force. Of course, the foregoing argument works only provided $s << r$. A more complete derivation can be given using vectors.

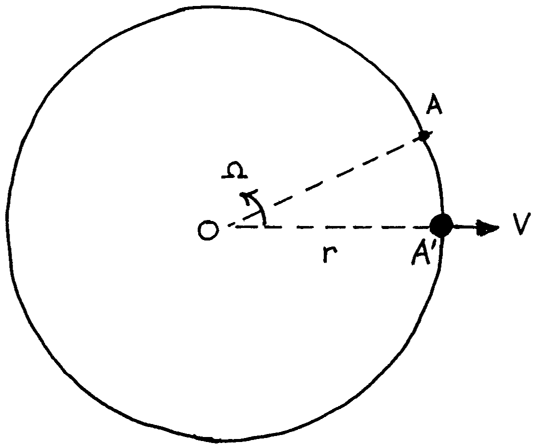


Figure 4.4. The deflection of a ball due to the Coriolis force.

If you think about it carefully, perhaps with the aid of a diagram, you will see that the ball will appear to you to curve to the right wherever you stand on the merry-go-round and in whichever direction you roll the ball. Moreover, if the merry-go-round rotates clockwise, the ball will appear to curve to the left under all circumstances.

Coriolis force in the atmosphere

The merry-go-round illustrates in a simple way why an additional force arises in a rotating frame of reference. However, the results are not directly transferable to motion in the atmosphere. We examine now, the meteorological role of the Coriolis force.

Consider a hypothetical particle of unit mass on the surface of the Earth. Ignore the effects of friction. Let **Ω** be the magnitude of the rotation of the Earth, R the distance from the axis of rotation to the particle, and u the eastward speed of the particle relative to the ground. Then the magnitude of the total centrifugal force felt by the particle is

$$\left(\Omega + \frac{u}{R}\right)^2 R = \Omega^2 + 2\Omega u + \frac{u^2}{R}. \qquad (4.4)$$

The first term on the right hand side is just the centrifugal force due to the rotation of the Earth. This is included in the definition of gravity. The second term is the Coriolis force, and has the same form as that derived for the merry-go-round problem. The third term is the contribution to the centrifugal force from the relative motion of the particle and can be neglected since $u << \Omega R$. The centrifugal force is, of course, a vector acting outward perpendicular to the axis of rotation with its magnitude given by Eq. (4.4). Thus, the components of the Coriolis force in the north-south and vertical directions are, respectively, $-2\Omega u \sin\phi$ and $2\Omega u \cos\phi$, where ϕ is the latitude.

For most scales of atmospheric motion, only the horizontal component of the Coriolis force is dynamically important (see Fig. 4.5). The *vertical* component of the Earth's angular velocity at latitude ϕ is $\Omega \sin \phi$. Then the *horizontal* component of Coriolis force per unit mass (or Coriolis acceleration) has magnitude fu where $f = 2\Omega \sin \phi$. The quantity f is called the *Coriolis parameter*. At the equator $\phi = 0$, whereupon $f = 0$ and the horizontal component of the Coriolis force is zero. The Coriolis force acts 90 degrees to the right of the wind (or to a ball!) in the northern hemisphere where the rotation of the Earth is counter-clockwise viewed from above the north pole (Fig. 4.5). Viewed from above the south pole, the Earth rotates clockwise and the Coriolis force acts to the left of the wind in the southern hemisphere.

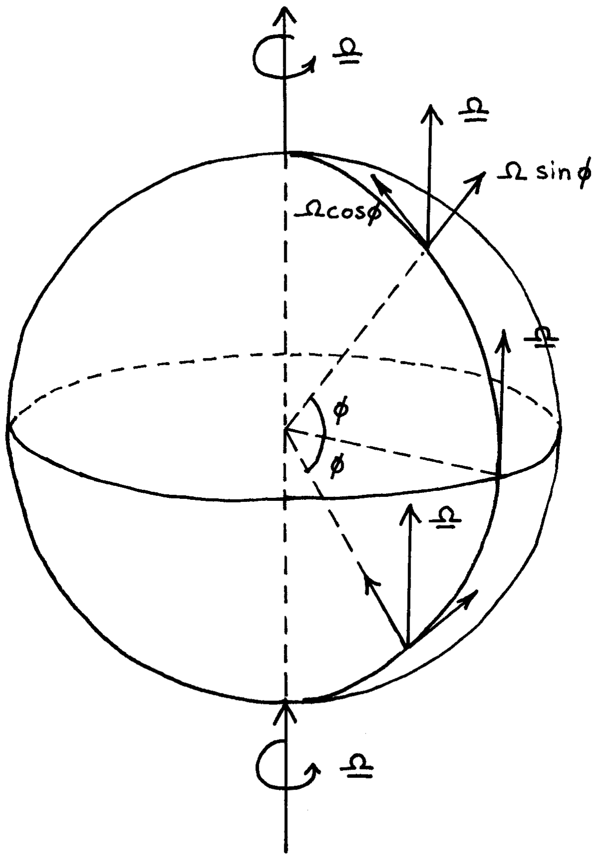


Figure 4.5. The variation of the horizontal and vertical components of the Coriolis parameter with latitude.

Meteorologists define rotation to be *cyclonic* when it is in the same sense as the Earth's rotation, and *anticyclonic* when it is opposite to that of the Earth. It follows that cyclonic rotation coincides with *anticlockwise* rotation in the northern hemisphere but with *clockwise* rotation in the southern hemisphere. Conversely, anticyclonic rotation is the same as clockwise flow in the northern hemisphere and anticlockwise flow in the southern hemisphere.

Geostrophic motion

To a good first approximation, the horizontal pressure gradient force balances the Coriolis force in large-scale weather systems such as extratropical cyclones and anticyclones. Motion that satisfies this condition is called *geostrophic.* The case in which the isobars are straight and parallel to the x-axis is illustrated in Fig. 4.6. Here,

$$0 = -\frac{1}{r}\frac{\partial p}{\partial y} - f u$$

or,
$$u = -\frac{1}{f r}\frac{\partial p}{\partial y} = u_g \qquad (4.5)$$

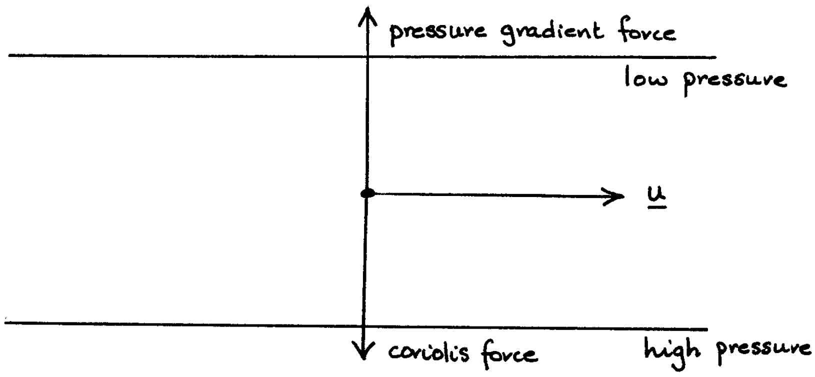


Figure 4.6. The balance of forces in geostrophic flow.

Equation (4.5) defines the *geostrophic wind*. In geostrophic motion, the wind blows parallel to the isobars with low pressure to the left in the northern hemisphere. In the southern hemisphere it blows with low pressure to the right since Ω, and hence the Coriolis force, have the opposite sign there. Note that in geostrophic flow there is no component of force in the flow direction and therefore no acceleration of the wind (see Fig. 4.6). The wind is approximately geostrophic for large-scale motion outside of the tropics and above the planetary boundary layer.

It follows from Eq. (4.5) that the *geostrophic wind* increases linearly with the pressure gradient. This accords with our experience of weather charts - the closer the isobars are together, the stronger is the wind. As we move towards the equator, *f* becomes smaller so that the same geostrophic wind speed can occur for a smaller pressure gradient. However, the geostrophic approximation breaks down at the equator where *f* is zero. Thus in equatorial regions, the flow is more down the pressure gradient (*i.e*., across the isobars towards low pressure) than along the isobars.

Returning to an earlier calculation, a pressure gradient of 10 mb/1000 km (or $10^{-3}m^2s^{-1}$) near the surface (ρ ÷ 1 kg/m^3) and at 45º latitude (f ÷$10^{-4}s^{-1}$) corresponds with a geostrophic wind of 10 ms^{-1} which is much more in line with observations.

East-west component of the Coriolis force

The argument given above gives the north-south component of the Coriolis force acting on a particle moving in the east-west direction. We consider now, the east-west component of the Coriolis force which acts when a particle moves with a component of its motion in the north-south direction. As a particle moves in the north-south direction it conserves its angular momentum. Since R increases as a particle moves equatorward, it must a relative westward component to its motion in order to conserve its angular momentum. Specifically, suppose a particle moves equatorward from latitude f_0 to latitude $f_0 + \Delta f$. If the particle is initially at rest relative to the Earth, conservation of angular momentum requires,

$$\Omega R^2 = \left(\Omega + \frac{\Delta u}{R + \Delta R} \right) (R + \Delta R) \qquad (4.6)$$

Expanding the right hand side of Eq. (4.6) and neglecting products of small quantities gives,

$$\Delta u = -2\Omega \Delta R = 2\Omega a_e \Delta f \sin f_0 \ ,$$

where $\Delta R = -a_e \Delta f \sin f_0$. Hence,

$$\frac{\Delta u}{\Delta t} = 2\Omega a_e \frac{\Delta f}{\Delta t} \sin f_0 \ .$$

Since, $v = a_e df / dt$ it follows that the component of the Coriolis force in the east-west direction is $f v$, the geostrophic wind is defined by

$$0 = -\frac{1}{r}\frac{\partial p}{\partial x} + f v$$

or,

$$v = \frac{1}{r}\frac{\partial p}{\partial x} = v_g \qquad (4.7)$$

5. Air Masses

A pioneering step in meteorological analysis was made in the early 1920's by Scandinavian meteorologists who introduced the idea of fronts and air masses to characterize the tropospheric structure in middle and high latitudes. It was found that large horizontal temperature gradients tended to be concentrated into narrow strips, perhaps a few hundred kilometres wide, and separated by larger regions of air with more uniform properties. The regions of large gradients were named fronts, while the regions of the atmosphere they separate were referred to as air masses. The concept of fronts and air masses and the charting of their distributions forms an important part of practical meteorology today. Air masses are characterized mainly by their surface temperatures and dew-point temperature, and by upper air temperature and humidity soundings.

The dew-point temperature, T_d, is defined as the temperature at which a parcel of air just becomes saturated as it is cooled isobarically. The 'dew point' as well as the 'dry-bulb' temperature is measured by conventional radiosondes. Moreover, $T - T_d$, the so-called dew-point depression gives a quick visual indication of how dry the air is at any level.

Fronts are sloping zones of relatively large temperature gradient that separate air masses. A simplified picture of the general circulation of the atmosphere shows a single front, the polar front, extending around the hemisphere with only slight modification and separating essentially polar air from tropical air. In the middle troposphere the polar front shows up clearly when isotherms are plotted in a hemispheric chart at 500 mb (Fig. 5.1). In the lower troposphere the temperature gradients at fronts are usually even stronger, but the polar front is not continuous around the hemisphere. The picture is more complicated for two reasons:

(i) vortical perturbations in the form of cyclones and anticyclones are capable of creating transitional air mass types and of forming individual detached fronts; and

(ii) the continents and oceans impart different properties to the overlying atmosphere and thus create contrasting air masses.

The formation and behaviour of fronts is a major topic in its own right. For the present we focus only on the air masses separated by fronts.

Source regions of air masses

Different air masses are created because certain sections of the atmosphere are acted upon for long periods of time (days and weeks) by radiative, convective and turbulent exchange processes, together with evaporation or condensation, characteristic of a particular region of the Earth. In the simplest picture described above, the air poleward of the polar front acquires characteristic of high latitudes and the air equatorwards has tropical or sub-tropical properties. Perturbations cause breaks in the front through which the air masses are mixed. Figure 5.2 provides a zonal mean view of the principal air masses as a function of latitude and height.

Regions in which air masses attain characteristic properties are called air mass source regions. Air masses are given names according to their source. In dealing with large sections of the Earth, however, it is not possible in practice to cling to the source name very long. Long trajectories of air masses over different parts of the Earth, which are to be expected as a consequence of the normal atmospheric circulation, subject the air masses to new source regions. Thus, the source name refers only to the recent history of the air mass.

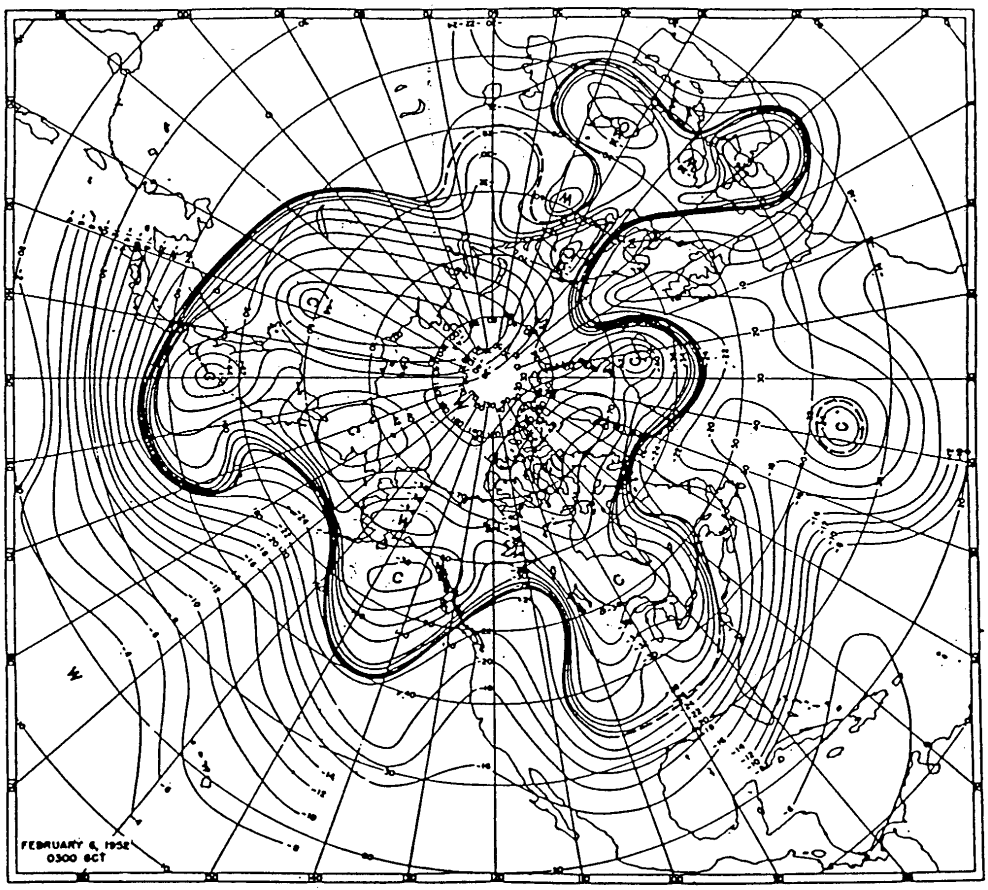


Figure 5.1. Temperature distribution at 500 mb of 6 February 1952; isotherms at 2^0C intervals. The heavy line marks the approximate southern limit of the polar air, including the frontal zone. (From Bradbury and Palmén, 1953).

In the lower troposphere the continental and maritime source regions stand in sharp contrast and during the extreme seasons of the year, summer and winter, produce air mass contrasts equal to or greater than the latitudinal effects. Another feature of the lower troposphere in the northern hemisphere is the appearance of a third air mass, artic air behind a second front.

With the combined effect of latitude and continents verses oceans, the general classification of air masses is as follows: maritime tropical (mT), continental tropical (cT), maritime polar (mP), and continental polar (cP).

In this context 'tropical' mean 'not polar'. The source of the tropical air mass may be anywhere equatorward of about 40° latitude. The prefix 'a' also is used, for arctic or antarctic air.

The air masses are identified, not only by the characteristic properties which they carry with them from the source region, but also by contrasts which appear between them along the fronts on the synoptic charts. Continuity in time between successive synoptic charts reveals the recent histories of the air masses.

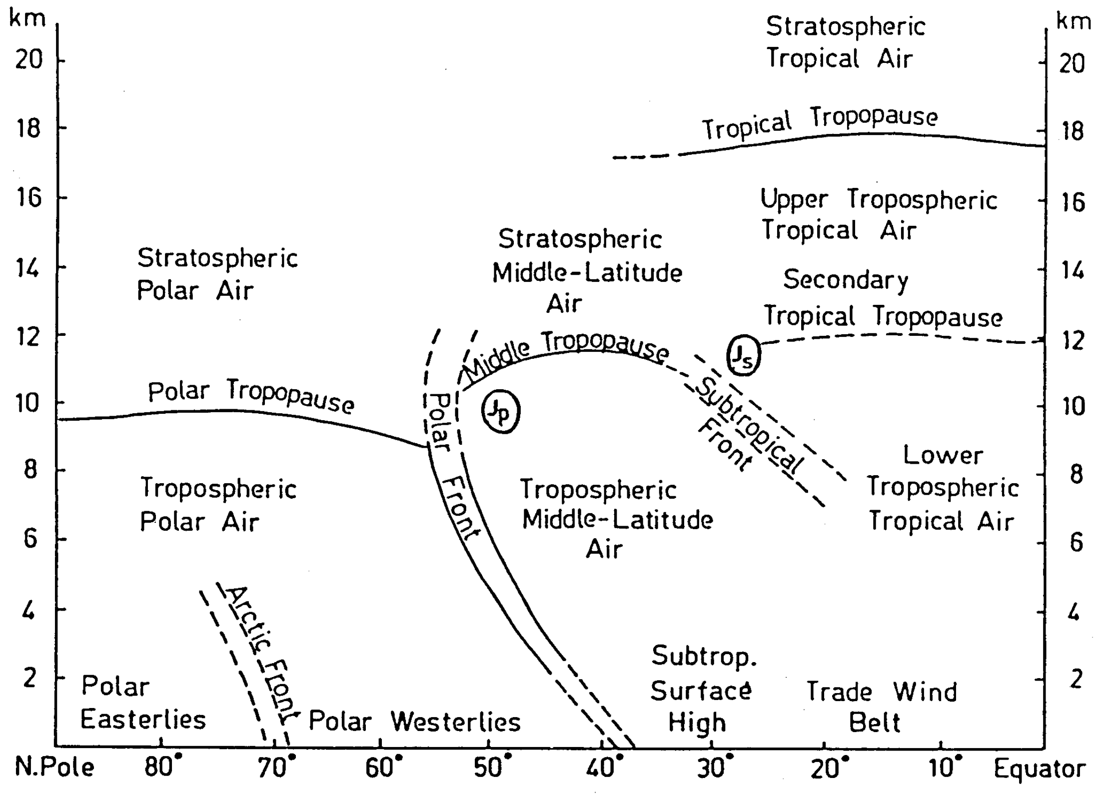


Figure. 5.2. The principal air masses, tropopauses and fronts, and jet streams in relation to the features of the low-level wind systems. Depending on the location and time, the fronts may be either well developed or relatively weak. (From Palmén and Newton, 1969).

Processes determining air-mass characteristics

As already implied, the air masses obtain their characteristics by radiation fluxes and by heat and water vapour fluxes through the air-earth boundary layers. The latitude and the nature of the underlying surface determine the relative importance of the various processes. The main types of modifying influences the air mass undergoes are sketched in Fig 5.3.

The thermal stability of the air limits the vertical transport of heat and other properties such as water vapour and haze, since the vertical exchange must be accomplished in the air parcels carried by vertical eddies of convective currents. If the temperature lapse rate is very stable, such as in the case of a temperature inversion, the radiative heat flux is of overriding importance (Fig. 5.3a). In the case of a moderate lapse rate, mechanical friction in the wind blowing across the surface of the Earth can set up eddies which will transport heat and other properties vertically. With superadiabatic or nearly adiabatic lapse rates, the mixing is aided by thermally-driven convection currents (Fig. 5.3a).

When air masses lie over a cold surface, the cooling from below creates a stable lapse rate, virtually cutting off vertical eddy exchange (Fig. 5.3b). The air mass is cooled almost entirely by radiative fluxes. Over a warm surface the heating from below creates a steep lapse rate in the low levels so that turbulent eddy exchange and convection carry the heat and water vapour quickly through a considerable depth of the air mass (Fig. 5.3a).

Cold air over a warm ocean will have heat and moisture transported quickly through a great height. When warm air moves over a cold ocean a surface-based temperature inversion is established. This inhibits turbulent mixing and further cooling must depend primarily on radiation fluxes to cool it further. Radiative processes proceed much more slowly than do the changes brought about by internal vertical motions over a warm surface.

It is found, however, that surface conditions alone do not determine the effects that will be produced on an air mass. Effects of ascent and subsidence are important in determining the lapse rates. In areas of strong cyclones and anti-cyclones, these dynamic processes may act more prominently than the thermal ones.

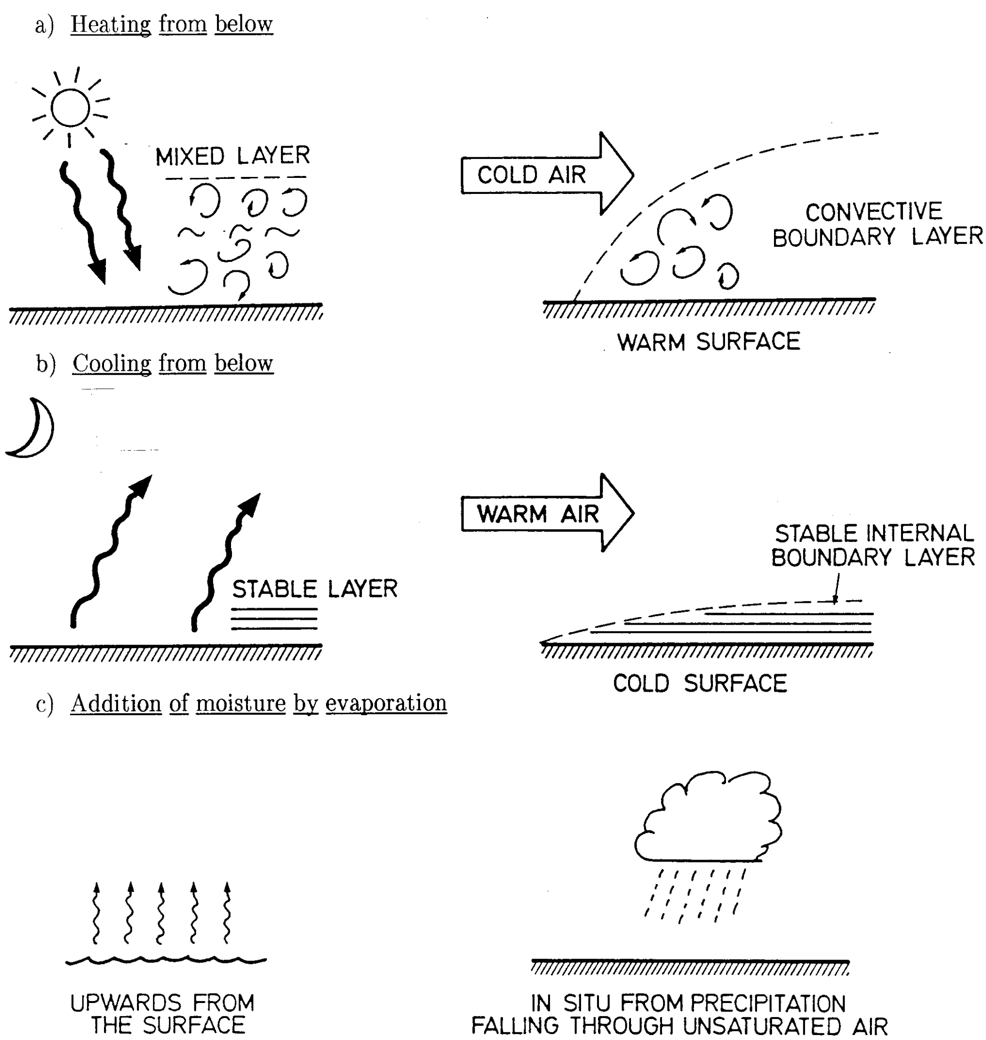


Figure 5.3. Some thermodynamic process of air mass modification.

A useful way of portraying changes in atmospheric structure at a particular place is to construct time-height cross-sections of various quantities. Figure 5.4 shows the isolines of constant potental temperature, called isentropes, at Mount Isa, during a period spanning the passage of a shallow cold front. The cross-section is drawn with time increasing to the left, so that, to the extent that the cold air mass is simply carried along (advected) without change in structure, the diagram suggests the cold front approaching from the west as actually occurred.

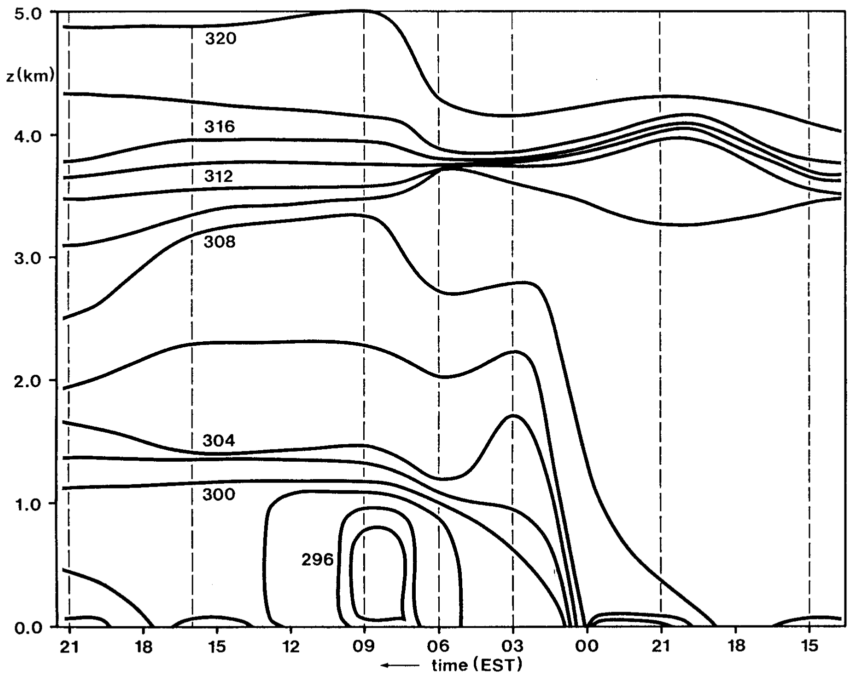


Figure 5.4. Time-height cross-section of potential temperature.

6. The Thermal Wind

One of the most prominent features of the atmosphere is that, on average, the wind increases with height. These increases in the wind speed with height are closely associated with horizontal gradients in temperature. For example, see Fig. 6.1.

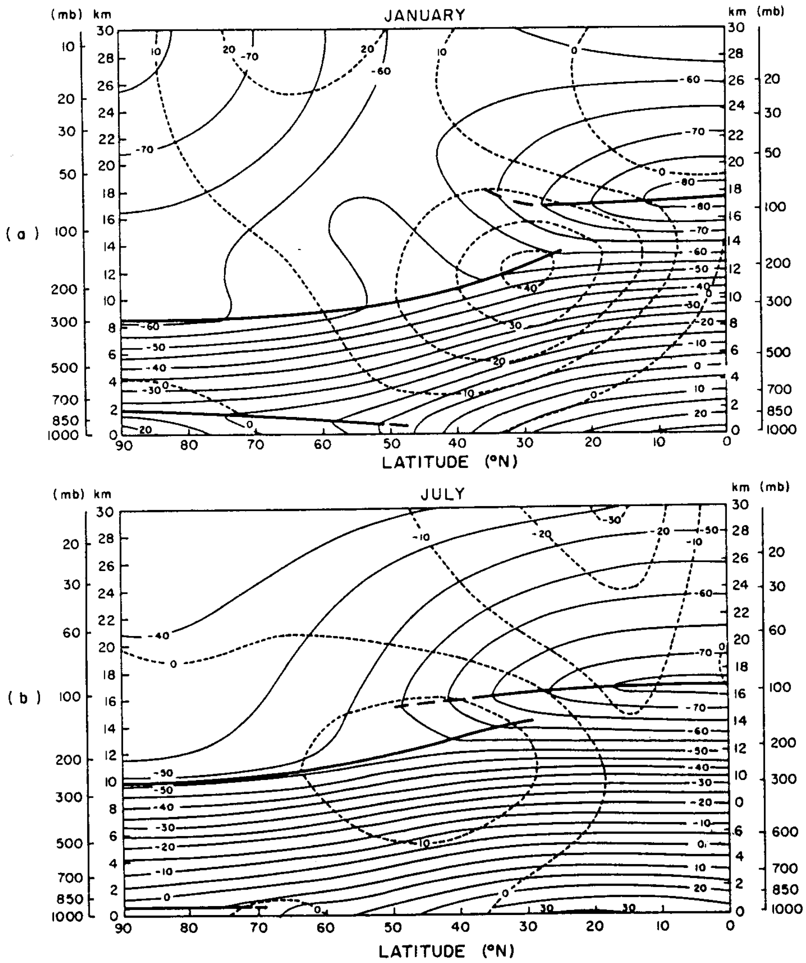


Figure 6.1. Meridional cross-section of longitudinally average temperature and zonal wind. (From Wallace and Hobbs, 1977.)

More on isobaric coordinates

From Fig. 6.2,

$$\frac{p_C - p_A}{\Delta x} = \frac{p_C - p_B}{\Delta x} + \frac{p_B - p_A}{\Delta x} = \frac{p_C - p_B}{\Delta z}\frac{\Delta z}{\Delta x} + \frac{p_B - p_A}{\Delta x}.$$

Since $p_C = p_A$,

$$\frac{p_B - p_A}{\Delta x} = -\frac{p_C - p_B}{\Delta z}\frac{\Delta z}{\Delta x}. \qquad (6.1)$$

Physically, $(p_B - p_A)/\Delta x$ is the horizontal pressure gradient, $(p_C - p_B)/\Delta z$ is the vertical pressure gradient, and $\Delta z/\Delta x$ is the slope of the pressure surface.

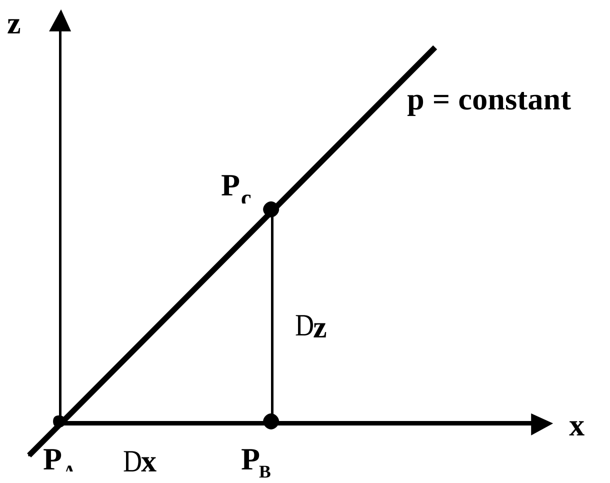


Figure 6.2. Relationship between gradients on a constant height surface and gradients on a constant pressure (isobaric) surface.

Consider now the limit as $\Delta x \to 0$ and $\Delta z \to 0$. Then, Eq. (6.1) becomes

$$\frac{\partial p}{\partial x} = \frac{\partial z}{\partial x}\left(-\frac{\partial p}{\partial z}\right) = r\,\frac{\partial \Phi}{\partial x}, \text{ since } -\frac{\partial p}{\partial z} = r\,g\,.$$

Note that $\partial z/\partial x$ is the change in height per change in horizontal distance along a pressure surface (i.e. *holding p constant*), and represents the slope of the pressure surface.

The horizontal pressure gradient force per unit mass in the x direction is

$$\frac{1}{r}\frac{\partial p}{\partial x} = \frac{\partial \Phi}{\partial x}. \qquad (6.2a)$$

On the left hand side the derivative is calculated holding z constant, which represents the change in pressure in the x –direction along a surface of constant height. The right hand side derivative is calculated with the pressure held constant, and represent the change of height in the x-direction following a pressure surface.

Similarly,

$$\frac{1}{r}\frac{\partial p}{\partial y} = \frac{\partial \Phi}{\partial y}. \qquad (6.2b)$$

Thus, the geostrophic wind in isobaric coordinates is

$$f\,v = \frac{1}{r}\frac{\partial p}{\partial x} = \frac{\partial \Phi}{\partial x} \quad \text{and} \quad f\,u = -\frac{1}{r}\frac{\partial p}{\partial y} = -\frac{\partial \Phi}{\partial y}. \qquad (6.3a,b)$$

The advantage of isobaric coordinates is that the density is eliminated. Thus maps of Φ at different pressure levels have the same scale for converting to velocities.

Thermal wind

Differentiating Eq. (6.3b) with respect to p gives

$$\begin{aligned} f\frac{\partial u}{\partial p} &= -\frac{\partial^2 \Phi}{\partial y\,\partial p} \\ &= \frac{\partial r^{-1}}{\partial y}, \text{ since } \frac{\partial \Phi}{\partial p} = -\frac{1}{r}, \qquad (6.4) \\ &= -\frac{1}{r^2}\frac{\partial r}{\partial y}. \end{aligned}$$

Here the differentiation is done holding p constant. If the density increases with y (as happens on average in the northern hemisphere), then u decreases with pressure. Sine the pressure decreases with height, u increases with height.

Similarly, $$f\frac{\partial v}{\partial p} = \frac{1}{r^2}\frac{\partial r}{\partial x}.$$

Alternatively, using the perfect gas law (Eq. 6.4),

$$f\frac{\partial u}{\partial p} = \frac{R_d}{p}\frac{\partial T}{\partial y} \quad \text{and} \quad f\frac{\partial v}{\partial p} = -\frac{R_d}{p}\frac{\partial T}{\partial x}.$$

Horizontal buoyancy (temperature) gradients in the atmosphere are associated with vertical gradients in the geostrophic wind. This *change* with height of the *geostrophic wind* is called the *thermal wind*. (Note that the term *thermal wind* is really a misnomer as it refers to a *gradient* in the wind and not the wind itself.) The reason that upper winds (above the friction layer) vary with height is that the isobaric surfaces are not in general parallel with the Earth's surface. In turn, this is a reflection of the existence of horizontal temperature gradients and the fact that pressure decreases with height more rapidly in cold air than in warm air. Thus, the geostrophic wind speed changes with height.

If there were no horizontal temperature gradient the vertical separation of any two isobaric surfaces would not vary in space. In this case the isobaric surfaces would be parallel to one another (though not necessarily straight), and the geostrophic wind would be independent of height.

There is a strong correlation between the strength of the horizontal temperature gradient and the vertical shear of the geostrophic wind. For example, both quantities have large values in the frontal zone and much smaller values in the warm air. In passing upward through the jet stream level, the horizontal temperature gradient and the vertical wind shear simultaneously undergo a reversal in sign. Hence, just above the jet stream level the temperature is usually higher on the poleward side of the jet stream than on the equatorward side.

$$f\frac{\partial u}{\partial z} = -\frac{g}{T}\frac{\partial T}{\partial y} \quad \text{and} \quad f\frac{\partial v}{\partial z} = \frac{g}{T}\frac{\partial T}{\partial x}.$$

The difference between gradients calculated on pressure surfaces and height surfaces is usually so small that the definition is unimportant for all practical purposes.

Using the definition of potential temperature,

$$f\frac{\partial u}{\partial z} = -\frac{g}{q}\frac{\partial q}{\partial y} \quad \text{and} \quad f\frac{\partial v}{\partial z} = \frac{g}{q}\frac{\partial q}{\partial x}.$$