

### Lecture 11. Surface Evaporation (Garratt 5.3)

The partitioning of the surface turbulent energy flux into sensible vs. latent heat flux is very important to the boundary layer development. Over ocean, SST varies relatively slowly and bulk formulas are useful, but over land, the surface temperature and humidity depend on interactions of the BL and the surface. How, then, can the partitioning be predicted?

For saturated ideal surfaces (such as saturated soil or wet vegetation), this is relatively straightforward. Suppose that the surface temperature is  $T_0$ . Then the surface mixing ratio is its saturation value  $q^*(T_0)$ . Let  $z_1$  denote a measurement height within the surface layer (e. g. 2 m or 10 m), at which the temperature and humidity are  $T_1$  and  $q_1$ . The stability is characterized by an Obukhov length  $L$ . The roughness length and thermal roughness lengths are  $z_0$  and  $z_T$ . Then Monin-Obukhov theory implies that the sensible and latent heat fluxes are

$$H_S = \rho c_p C_H V_1 (T_0 - T_1),$$

$$H_L = \rho L C_H V_1 (q_0 - q_1), \quad \text{where } C_H = \text{fn}(V_1, z_1, z_0, z_T, L)$$

We can eliminate  $T_0$  using a linearized version of the Clausius-Clapeyron equations:

$$q_0 - q^*(T_1) = (dq^*/dT)_R (T_0 - T_1), \quad R \text{ indicates a value at a reference temperature,}$$

that ideally should be close to  $(T_0 - T_1)/2$

$$H_L = s^* H_S + \rho L C_H V_1 (q^*(T_1) - q_1), \quad s^* = (L/c_p)(dq^*/dT)_R (= 0.7 \text{ at } 273 \text{ K, } 3.3 \text{ at } 300 \text{ K}) \quad (1)$$

This equation expresses latent heat flux in terms of sensible heat flux and the saturation deficit at the measurement level. It is immediately apparent that the Bowen ratio  $H_S/H_L$  must be at most  $s^{*-1}$  over a saturated surface, and that it drops as the relative humidity of the overlying air decreases. At higher temperatures, latent heat fluxes tend to become more dominant. For an ideal surface, (1), together with energy balance

$$R_N - H_G = H_S + H_L$$

can be solved for  $H_L$ :

$$H_L = LE_P = \Gamma(R_N - H_G) + (1 - \Gamma)\rho L C_H V_1 (q^*(T_1) - q_1) \quad (2)$$

$$\Gamma = s^* / (s^* + 1) (= 0.4 \text{ at } 273 \text{ K, } 0.77 \text{ at } 300 \text{ K})$$

The corresponding evaporation rate  $E_P$  is called the **potential evaporation**, and is the maximum possible evaporation rate given the surface characteristics and the atmospheric state at the measurement height. If the surface is not saturated, the evaporation rate will be less than  $E_P$ . The figure on the next page shows  $H_L$  vs. the net surface energy influx  $R_N - H_G$  for  $T_1 = 293 \text{ K}$  and  $\text{RH}_1 = 57\%$ , at a height of  $z_1 = 10 \text{ m}$ , with a geostrophic wind speed of  $10 \text{ m s}^{-1}$ , assuming a range of surface roughness. Especially over rough surfaces (forest),  $H_L$  often exceeds  $R_N - H_G$ , so the sensible heat flux must be negative by up to  $100 \text{ W m}^{-2}$ . The Bowen ratio is quite small (0.2 or less) for all the saturated surfaces shown in this figure.

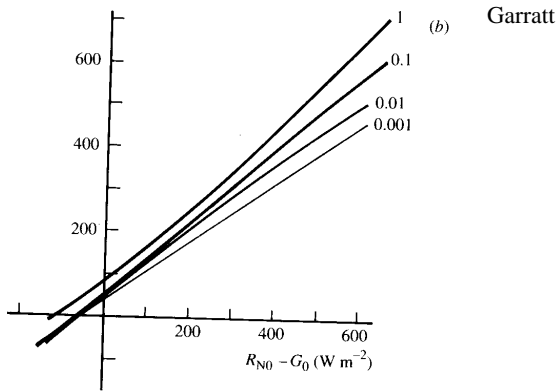


Fig. 5.6 Potential evaporation for different wet surfaces calculated from Eq. 5.26. In (a) neutral conditions have been assumed, and in (b) the full stability correction in  $r_{av}$  is included (see Eqs. 3.47 and 3.57). Note how the effects of thermal stability tend to reduce the direct influence of aerodynamic roughness. Values of  $z_0$  are as follows: 0.001 m, lake; 0.01 m, grass; 0.1 m, scrub; 1 m, forest. Further details of the calculations can be found in Webb (1975).

### Evaporation from dry vegetation

We consider a fully vegetated surface with a single effective surface temperature and humidity (a ‘single-layer canopy’). The sensible heat flux is originates at the leaf surfaces, whose temperature is  $T_0$ . The latent heat flux is driven by evaporation of liquid water out of the intercellular spaces within the leaves through the stomata, which are channels from the leaf interior to its surface. The evaporation is proportional to the humidity difference between the saturated inside of the stomata and the ambient air next to the leaves. The constant of proportionality is called the **stomatal resistance** (units of inverse velocity)

$$r_{st} = \rho(q^*(T_0) - q_0)/E \quad (3)$$

Plants regulates transport of water vapor and other gasses through the stomata to maintain an optimal internal environment, largely shutting down the stomata when moisture-stressed. Hence  $r_{st}$  depends not only on the vegetation type, but also soil moisture, temperature, etc. Table 5.1 of Garratt shows measured  $r_{st}$ , which varies form 30 -300  $s\ m^{-1}$ .

By analogy, we can define an **aerodynamic resistance**

$$r_a = (C_H V_1)^{-1} = \rho(q_0 - q_1)/E \quad (4)$$

Typical values of  $r_a$  are 100  $s\ m^{-1}$ , decreasing in high wind or highly convective conditions. This is comparable to the stomatal resistance. Working in terms of aerodynamic resistance in place of  $C_H$  is convenient in this context, as we shall see next, because these resistances add:

$$r_{st} + r_a = \rho(q^*(T_0) - q_0)/E + \rho(q_0 - q_1)/E = \rho(q^*(T_0) - q_1)/E, \quad (5)$$

i. e.  $E$  is identical to the evaporation rate over an equivalent saturated surface with aerodynamic resistance  $r_{st} + r_a$ . The same manipulations that led to (1) and (2) now lead to:

$$\begin{aligned} H_S &= \rho c_p (T_0 - T_1) / r_a \\ H_L &= LE = \rho(q^*(T_0) - q_1) / (r_{st} + r_a) = \{s^* H_S + \rho L(q^*(T_1) - q_1)\} \{r_a / (r_{st} + r_a)\} \\ H_L &= \Gamma^*(R_N - H_G) + (1 - \Gamma^*) \rho L(q^*(T_1) - q_1) / (r_{st} + r_a) \end{aligned} \quad (6)$$

where  $\Gamma^* = s^*/(s^* + 1 + r_{st}/r_a)$

This is the **Penman-Monteith** relationship. Comparing (6) to (2), we find that  $\Gamma^* < \Gamma$ , so the heat flux will be partitioned more into sensible heating, especially if stomatal resistance is high, winds are high, or the BL is unstable. The effect is magnified at cold temperatures where  $s^*$  is small. The ratio of  $H_L$  to the saturated latent heat flux (2) given the same energy influx  $R_N - H_G$  is

$$H_L/H_{L, sat} = \frac{1}{1 + (1 - \Gamma)(r_{st}/r_a)}$$

Calculations of this ratio for neutral conditions, a  $10 \text{ m s}^{-1}$  geostrophic wind speed, and various surface roughnesses are shown in the figure below. For short grass, the surface transfer coefficient is low, so the aerodynamic resistance is high and stomatal resistance does not play a crucial role at high temperatures (though at low temperatures it cuts off a larger fraction of the latent heat flux). For forests, stomatal resistance is very important due to the high surface roughness (low aerodynamic resistance).

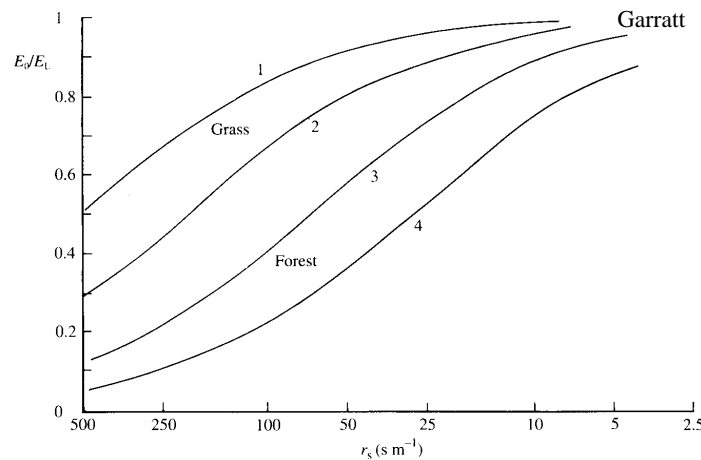


Fig. 5.8 Variations of  $E_0/E_L$  (Eq. 5.37) with surface resistance. Values of  $r_{av}$  have been calculated for neutral conditions, with  $z_g = z_0/7.4$ . For short grass ( $z_0 = 0.0025 \text{ m}$ ): curve 1,  $T = 303 \text{ K}$ ; curve 2,  $T = 278 \text{ K}$ . For forest ( $z_0 = 0.75 \text{ m}$ ): curve 3,  $T = 303 \text{ K}$ ; curve 4,  $T = 278 \text{ K}$ .

### Soil moisture

If the surface is partly or wholly unvegetated, the evaporation rate depends on the available soil moisture. Soil moisture is also important because it modulates the thermal conductivity and hence the ground heat flux, and affects the surface albedo as well as transpiration by surface vegetation. For instance, Idso et al. (1975) found that for a given soil, albedo varied from 0.14 when the soil was moist to 0.31 when it was completely dry at the surface.

If the soil-surface relative humidity  $RH_0$  is known, then the evaporation is

$$E = \rho(RH_0q^*(T_0) - q_1)/r_a .$$

Note that net evaporation ceases when the mixing ratio at the surface drops below the mixing ratio at the measurement height, which does not require the soil to be completely dry. Soil moisture can be expressed as a volumetric moisture content  $\eta$  (unitless), which does not exceed a saturated value  $\eta_s$ , usually around 0.4. When the soil is saturated, moisture can easily flow through it, but not all pore spaces are water-filled. As the soil becomes less saturated, water is increasingly bound to the

soil by adsorption (chemicals) and surface tension.

The movement of water through the soil is down the gradient of a combined gravitational potential  $gz$  (here we take  $z$  as depth below the surface) plus a moisture potential  $g\psi(\eta)$ . The moisture potential is always negative, and becomes much more so as the soil dries out and its remaining water is tightly bound. Note  $\psi$  has units of height. The downward flux of water is

$$F_w = -\rho_w K(\eta) \partial(\psi + z) / \partial z, \quad (\text{Darcy's law})$$

where  $K(\eta)$  is a hydraulic conductivity (units of  $\text{m s}^{-1}$ ), which is a very rapidly increasing function of soil moisture. Conservation of soil moisture requires

$$\rho_w \partial \eta / \partial t = -\partial F_w / \partial z$$

The surface relative humidity is

$$\text{RH}_0 = \exp(-g\psi|_{z=0} / R_v T_0)$$

i. e. the more tightly bound the surface moisture is to the soil, the less it is free to evaporate. Empirical forms for  $\psi$  and  $K$  as functions of  $\eta$  have been fitted to field data for various soils:

$$\psi = \psi_s (\eta / \eta_s)^{-b}$$

$$K = K_s (\eta / \eta_s)^{2b+3}$$

where  $\psi_s$  and  $K_s$  are saturation values, depending on the soil, and the exponent  $b$  is 4-12. For  $b = 5$ , halving the soil moisture increases the moisture potential by a factor of 32 and decreases the hydraulic conductivity by a factor of 4000! Because these quantities are so strongly dependent on  $\eta$ , one can define a critical surface soil moisture, the wilting point  $\eta_w$ , above which the surface relative humidity  $\text{RH}_0$  is larger than 99%, and below which it rapidly drops. The wilting point can be calculated as the  $\eta$  below which the hydraulic suction  $-\psi$  exceeds 150 m.

Garratt

Table A9. Soil moisture quantities for a range of soil types, based on Clapp and Hornberger (1978)

Quantities shown are as follows:  $\eta_s$  is the saturation moisture content (volume per volume),  $\eta_w$  is the wilting value of the moisture constant which assumes 150 m suction (i.e. the value of  $\eta$  when  $\psi = -150$  m),  $\psi_s$  is the saturation moisture potential and  $K_{\eta_s}$  is the saturation hydraulic conductivity;  $b$  is an index parameter (see Eqs. 5.46-5.48).

Soil type	$\eta_s$ ( $\text{m}^3 \text{m}^{-3}$ )	$\psi_s$ (m)	$K_{\eta_s}$ ( $10^{-6} \text{m s}^{-1}$ )	$b$	$\eta_w$ ( $\text{m}^3 \text{m}^{-3}$ )
1. sand	0.395	- 0.121	176	4.05	0.0677
2. loamy sand	0.410	- 0.090	156.3	4.38	0.075
3. sandy loam	0.435	- 0.218	34.1	4.90	0.1142
4. silt loam	0.485	- 0.786	7.2	5.30	0.1794
5. loam	0.451	- 0.478	7.0	5.39	0.1547
6. sandy clay loam	0.420	- 0.299	6.3	7.12	0.1749
7. silty clay loam	0.477	- 0.356	1.7	7.75	0.2181
8. clay loam	0.476	- 0.630	2.5	8.52	0.2498
9. sandy clay	0.426	- 0.153	2.2	10.40	0.2193
10. silty clay	0.492	- 0.490	1.0	10.40	0.2832
11. clay	0.482	- 0.405	1.3	11.40	0.2864

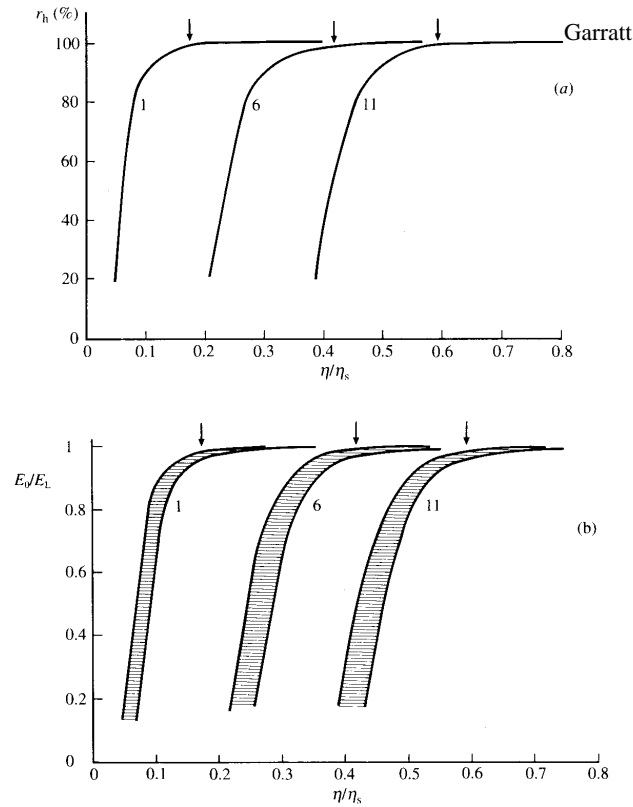


Fig. 5.9 (a) Relative humidity  $r_h$  as a function of relative soil moisture content  $\eta/\eta_s$ , based on Eq. 5.49 and data in Table A9 for soil types 1 (sand), 6 (loam) and 11 (clay). Calculations are for a temperature  $T_0$  of 303 K. The vertical arrows indicate the wilting points. Note that combining Eqs. 5.46 and 5.49 allows  $r_h$  to be calculated from  $\ln r_h = -(g/R_v T_0) \psi_s (\eta/\eta_s)^{-b}$ . (b)  $E_0/E_L$  as a function of the relative soil moisture content, based on numerical simulations in an atmospheric model for a range of climate conditions (mid-latitude summer) represented by the shaded regions (the temperature range is 283–303 K and  $q = 0.005$ ).

Landuse Integer Identification	Landuse Description	Albedo (%)		Moisture Avail. (%)		Emissivity (% at 9 μm)		Roughness Length (cm)		Thermal Inertia (cal cm <sup>-2</sup> K <sup>-1</sup> s <sup>-1/2</sup> )	
		Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win
1	Urban land	18	18	5	10	88	88	50	50	0.03	0.03
2	Agriculture	17	23	30	60	92	92	15	5	0.04	0.04
3	Range-grassland	19	23	15	30	92	92	12	10	0.03	0.04
4	Deciduous forest	16	17	30	60	93	93	50	50	0.04	0.05
5	Coniferous forest	12	12	30	60	95	95	50	50	0.04	0.05
6	Mixed forest and wet land	14	14	35	70	95	95	40	40	0.05	0.06
7	Water	8	8	100	100	98	98	.0001	.0001	0.06	0.06
8	Marsh or wet land	14	14	50	75	95	95	20	20	0.06	0.06
9	Desert	25	25	2	5	85	85	10	10	0.02	0.02
10	Tundra	15	70	50	90	92	92	10	10	0.05	0.05
11	Permanent ice	55	70	95	95	95	95	5	5	0.05	0.05
12	Tropical or sub-tropical forest	12	12	50	50	95	95	50	50	0.05	0.05
13	Savannah	20	20	15	15	92	92	15	15	0.03	0.03

MM5 surface types and their characteristics (Appendix 4 of MM5 manual)

*Parameterization of surface evaporation in large-scale models*

In practice, simplified formulations of soil moisture and transpiration are used in most models. We will defer most of these until later. However, the MM5 formulation of surface evaporation is particularly simplified. It is

$$E = \rho L C_H V_1 M (q^*(T_0) - q_1),$$

i. e. the standard formula for evaporation off a *saturated* surface at the ground temperature  $T_0$  (calculated by the model) multiplied by a moisture availability factor  $M$  between 0 and 1 that is assumed to depend only on the surface type. This formulation avoids the need to initialize soil moisture, but is tantamount to assuming a surface resistance that is proportional to the aerodynamic resistance, with

$$r_s/r_a = (1 - M)/M$$

While this type of formulation can be tuned to give reasonable results on an annually averaged basis, it is likely to be in error by a factor of two or more in individual situations, because  $r_s$  and  $r_a$  are both subject to large and independent fluctuations. More sophisticated schemes explicitly prognose soil moisture (often using relaxation to specified values deep within the soil to control fluctuations) and vegetation characteristics and determine the evaporation from these.