Lecture13. The stable BL (Garratt 6.2)

The stable nocturnal BL (NBL) has proved one of the more difficult types of BL to understand and model. The boundary layer tends to be only 50-300 m deep. Turbulence tends to be intermittent and gravity-wave like motions are often intermingled with turbulence, especially in the upper part of the boundary layer. Radiative cooling in the air often has a comparable effect on the stratification to the turbulence itself, reaching 1 K hour^{-1} or more in the lowest 50-100 m (by comparison, a downward heat flux of H_0 = -10 W m^{-2} out of a NBL h = 100 m would cool it at a rate $dq/dt)_{turb}$ = $H_0/\rho c_p h$ = 10^{-4} K s^{-1} = 0.3 K hr^{-1}. Even the largest turbulent eddies do not span the entire BL so there is a tendency to layering of chemicals and aerosols within the BL, especially in the upper part of the BL where turbulence is weakest. Wind profiles are much less well-mixed at night than during the daytime convective BL.

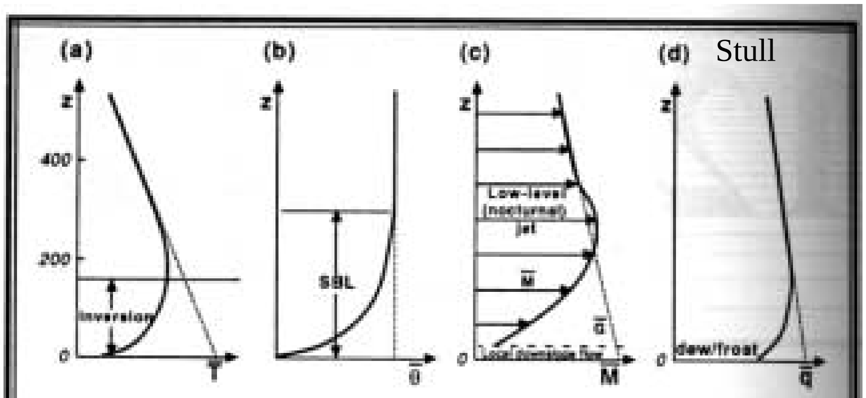


Fig. 12.1 Typical SBL profiles of mean (a) absolute temperature, (b) potential temperature, (c) wind speed, and (d) specific humidity.

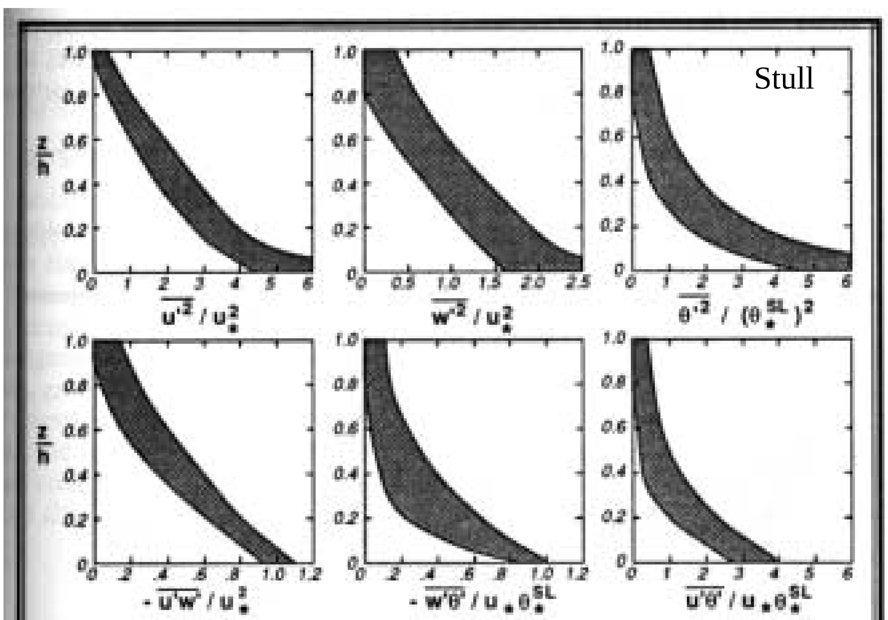


Fig. 12.3 Vertical profiles of dimensionless variances and covariances in the SBL. Shading indicates spread of data. (After Caughey; et al., 1979).

In an NBL, turbulence decreases sharply with height.

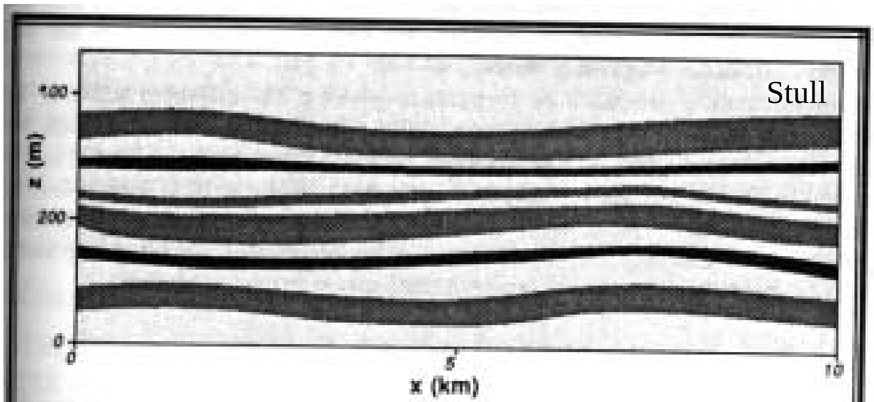
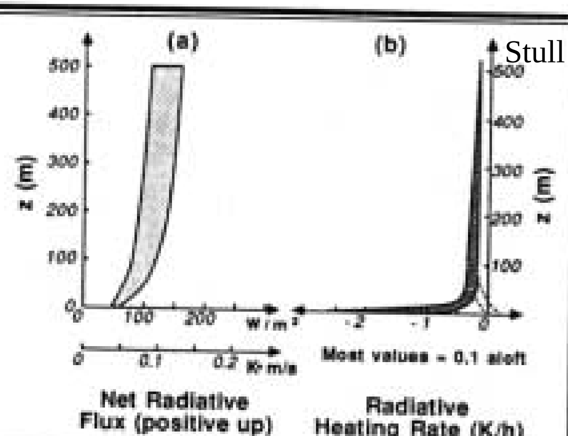


Fig. 12.2 Sketch of the layered aerosol structure often seen by lidar in SBLs. (After Eloranta, personal communication, 1987).

Layered NBL with gravity wave undulations that can modulate local shear, stratification, and hence turbulence.

Fig. 12.6 Variation of (a) net radiation within the SBL, and (b) the resulting radiative cooling rate. Some investigators suggest that there is heating, rather than cooling, in the air near the ground (shown dashed in (b)). Garratt and Brost, 1981; Turton and Brown, 1987; Cerni and Parish, 1984; Andre and Mahrt, 1982; Carlson and Stull, 1986; Stull, 1983.



Near a cold surface, radiative cooling can be surprisingly fast and helps maintain a stable stratification.

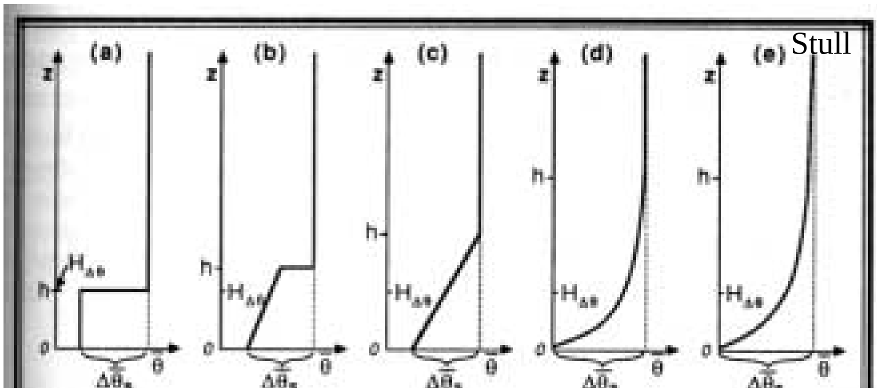


Fig. 12.5 Idealized models of SBL potential temperature profile. (a) well-mixed; (b) linearly-mixed; (c) linear; (d) polynomial; (e) exponential.

(b) typifies strong-wind NBL, (d) a weak-wind NBL under clear sky.

An idealized NBL model

One illuminating theoretical idealization is a NBL of constant depth driven by surface cooling only (Nieuwstadt 1984, *J. Atmos. Sci.*, **41**, 2202-2216). In practice, this is most realistic when winds are strong, producing sufficient turbulence to make substantial downward buoyancy fluxes that are much larger than the radiative flux divergence across the NBL (which is typically less than 10 W m^{-2}). We take the friction velocity u_*, the geostrophic wind U_g (taken to be in the +x direction), and the Coriolis parameter f as given. (In a practical application we would likely know the surface roughness length z_0, not u_*, but we could use the solution below to relate these two parameters). We assume:

(i) The entire BL, extending up to a fixed but unknown height h, is cooling at the same rate, and maintains fixed vertical profiles of stratification and wind.

(ii) No turbulence at the top of the BL

(iii) Within the bulk of the BL (above the surface layer), the sink of TKE due to buoyancy fluxes is assumed to be a fixed fraction Rf ≈ 0.2 of the shear production of TKE. The remaining fraction (0.8) of the shear-produced TKE goes to turbulent dissipation, as transport is observed to be negligible. This is the same as saying that the flux Richardson number Rf = 0.2.

(iv) No radiative cooling within the BL

(v) The (unknown) Obuhkov length L is assumed much smaller than the boundary layer depth. Hence, the largest eddies have a depth which is order of L, since deeper eddies do not have enough TKE to overcome the stratification by the scaling arguments we made in discussing the z-less scaling at $z \gg L$ when we discussed Monin-Obhukov theory.

(vi) The eddies act as an unknown, height-dependent eddy viscosity and diffusivity $K_m = K_h$ as suggested by Monin-Obuhkov theory. Hence the gradient Richardson number Ri = Rf, so is also 0.2 throughout the BL.

(vii) The BL is barotropic.

Scaling

Note that one could also use first-order closure on this problem instead of invoking assumptions (iii), (v) and (vi) about the eddies and their transports. This would give a largely similar answer as long as the lengthscale in the first-order closure was on the order of L through most of the boundary layer depth, and could also be used to relax the assumptions of steadiness, uniform cooling rate, no radiative cooling, and no thermal wind. However, the equations would not permit a closed-form solution which displays the parametric dependences clearly. We first scale the steady-state momentum equations, then use a clever approach to solve them.

Assumptions (i) and (ii) imply that if the (unknown) surface buoyancy flux is $B_0 < 0$, then

$$B(z) = \overline{w'b'} = B_0(1 - z/h) \tag{1}$$

The steady-state BL momentum equations are

$$-f(v - v_g) = -\partial/\partial z(\overline{u'w'}) \tag{2}$$

$$f(u - u_g) = -\partial/\partial z(\overline{v'w'}) \tag{3}$$

If {} indicates 'scale of', the above assumptions imply:

$$\{u'\} = \{v'\} = \{w'\} = u_*$$

$$\{K_m\} = \{\text{eddy velocity scale}\}\{\text{eddy lengthscale}\} = u_* L$$

$$\Rightarrow \{\partial u/\partial z\} = \{\overline{u'w'}\}/\{K_m\} = u_*^2/u_*L = u_*/L \text{ (similarly for } v)$$

$$\{\partial/\partial z\} = h^{-1}$$

To apply this scaling to (2)-(3), we differentiate them with respect to z, noting that the geostrophic wind is constant with respect to height by assumption (vii):

$$-f\partial v/\partial z = -\partial^2/\partial z^2(\overline{u'w'}) \quad (4)$$

$$f\,\partial u/\partial z = -\,\partial^2/\partial z^2(\overline{v'w'}) \quad (5)$$

Scaling the two sides of (4), we find

$$\{f\partial v/\partial z\} = fu_*/L = \{\partial^2/\partial z^2(\overline{u'w'})\} = u_*^2/h^2.$$

The same scaling holds for (5). This implies a scaling for BL depth h:

$$h = \gamma_c(u_*L/f)^{1/2} \quad (6)$$

where γ_c is an as yet unknown proportionality constant.

<u>Solution</u>

Now we have understood the scaling of the equations, we solve them in nondimensional form. This is a bit technical, so feel free to skip to the results. It is mathematically advantageous to combine (4) and (5) into one nondimensional complex-valued equation. Let the nondimensional height, shear, momentum flux and eddy viscosity be:

$$\xi = z/h, \quad s_v = (L/u_*)\,\partial(u + iv)/\partial z \text{ and } \sigma = (\overline{u'w'} + i\,\overline{v'w'})/u_*^2, \quad \kappa_m(\xi) = K_m/u_*L$$

Then (4) and (5) can be written:

$$s_v = i\gamma_c^{-2}\,\partial^2\sigma/\partial\xi^2 \quad (7)$$

The boundary conditions come from the definition of friction velocity, and assumption (ii) that stress vanish at the BL top. The surface momentum flux u_*^2 is in the direction opposite the wind. If the (unknown) surface cross-isobaric wind turning angle is α, then the two BCs are:

$$\sigma(0) = -e^{i\alpha}(1 + 0i)$$

$$\sigma(1) = 0 + 0i$$

The eddy viscosity assumption (vi) implies that $\sigma = -\kappa_m(\xi)s_v$. Since the nondimensional eddy viscosity is real, this is equivalent to requiring that the complex numbers σ and s_v. have opposite phase at all nondimensional heights s_v.

The last condition we must enforce is (iii), that buoyant consumption of TKE is 0.2 of shear production:

$$0.2 = \mathrm{Rf} = -B(z)/S(z) = -B_0(1 - z/h)/(\overline{u'w'}\partial u/\partial z + \overline{v'w'}\partial v/\partial z)$$

$$= -B_0(1 - \xi) / (u_*^3/L)\mathrm{Re}(\sigma^* s_v) \quad \text{(* denotes complex conjugate)}$$

Substituting (7) in for s_v, noting that by definition of Obuhkov length, $-B_0 = u_*^3/kL$, and that the eddy viscosity assumption implies that $\sigma^* s_v$ is guaranteed to be real, we obtain the nonlinear ODE

$$\sigma^*\partial^2\sigma/\partial\xi^2 = i\lambda(1 - \xi), \text{ where } \lambda = \gamma_c^2/k\text{Rf} \text{ is unknown} \quad (8)$$

This equation can be solved systematically by substituting $\sigma = re^{i\theta}$ and obtaining a pair of ODEs for $r(\xi)$ and $\theta(\xi)$. However, an easier approach is to look for a trial solution in the form

$$\sigma(\xi) = -e^{i\alpha}(1 - \xi)^a$$

This solution automatically obeys the boundary conditions and has the right form to match the RHS of (10). Substituting into (10), we find that this trial solution works if

$$a^* + a - 2 = 1$$

$$a(a - 1) = i\lambda$$

Setting $a = a_r + ia_i$, the first of these equations implies that $a_r = 3/2$. From the second, we deduce that

$$0 = \text{Re}[a(a - 1)] = \text{Re}[(3/2 + ia_i)(1/2 + ia_i)] = 3/4 - a_i^2 \Rightarrow a_i = 3^{1/2}/2$$

$$\Rightarrow \sigma(\xi) = -e^{i\alpha}(1 - \xi)^{(3 + \sqrt{3})/2},$$

$$\lambda = \text{Im}[a(a - 1)] = \text{Im}[(3/2 + ia_i)(1/2 + ia_i)] = 2a_i = 3^{1/2} = \gamma_c^2/k\text{Rf}$$

$$\Rightarrow \gamma_c = [3^{1/2}k\text{Rf}]^{1/2} = 0.37 \quad \text{so} \quad h = 0.37(u_*L/f)^{1/2},$$

$$\Rightarrow s_v = i\gamma_c^{-2}\partial^2\sigma/\partial\xi^2 = -ia(a-1)\gamma_c^{-2}e^{i\alpha}(1 - \xi)^{(-1 + i\sqrt{3})/2}$$

$$= \lambda\gamma_c^{-2}e^{i\alpha}(1 - \xi)^{(-1 + i\sqrt{3})/2} \text{ is non-dim shear.}$$

$$\Rightarrow \kappa_m(\xi) = -\sigma/s_v = (1 - \xi)^2\gamma_c^2/\lambda = 0.08(1 - \xi)^2 \text{ is non-dim eddy viscosity}$$

Hence, remarkably we have been able to deduce the BL depth. There is one shortcoming, which is that L must still be deduced. The deduced eddy viscosity decreases with height to zero at the BL top, as we'd expect since turbulence is concentrated at the surface. The shear profile can be integrated from $\xi = 1$ ($z = h$) and the resulting velocity profile redimensionalized to obtain:

$$u + iv\Big|_h^z = -[u_*h/L][2\lambda\gamma_c^{-2}/(1 + i\sqrt{3})]\, e^{i\alpha}(1 - \xi)^{(1 + i\sqrt{3})/2}$$

At the BL top, the velocity is U_g. At the surface, the velocity is zero. Hence, setting $\xi = 0$ on the RHS, and noting that $(1 + i\sqrt{3})/2 = \exp(i\pi/3)$ and that we have:

$$u + iv\Big|_h^0 = -U_g = -[u_*h/L][\lambda\gamma_c^{-2}]\exp(i\alpha - i\pi/3) \quad (9)$$

For consistency the RHS must be real, and have the same magnitude as the LHS. Thus

$$\alpha = \pi/3 \quad \text{(surface isobaric wind turning angle of 60 degrees)} \quad (10)$$

$$U_g = [u_*h/L][\lambda\gamma_c^{-2}] = [u_*h/L][1/k\,\text{Rf}] \quad (11)$$

Summary of Results and Comparison to Observations

PBL depth $h = \gamma_c(u_*L/f)^{1/2}$, where $\gamma_c = (3^{1/2}k\text{Rf})^{1/2} = 0.37$

Wind profile $(u + iv)/U_g = 1 - (1 - z/h)^{(1 + i\sqrt{3})/2}$

Note that h can be expressed in terms of the given parameters as:

$$h = 0.37(u_*L/f)^{1/2} = 0.37(-u_*^4/kB_0f)^{1/2} = 0.37(-u_*^4/0.12kf^2U_g^2)^{1/2} = 1.7u_*^2/fU_g$$

A larger friction velocity, smaller geostrophic wind, or lower latitude will increase h. Also note that the wind profile is independent of the surface roughness (except in the surface layer $z << L$, where the assumed eddy scale of L is no longer applicable and (12) is invalid). The surface isobaric turning angle is 60 degrees, and the wind turns to geostrophic at the PBL top. We can solve (11) for the

Obhukov length $L = h\,(u_*/U_g)(1/k\,\mathrm{Rf}) = 12.5h(u_*/U_g)$

Substituting for h, this can also be written as

$$L = \gamma_c(u_*L/f)^{1/2}\,(u_*/U_g)(1/k\,\mathrm{Rf})$$

or

$$L = (u_*^3/fU_g^2)(\gamma_c/k\,\mathrm{Rf})^2$$

This can be used to deduce the surface buoyancy flux, which by definition of L is:

Surface buoyancy flux $B_0 = -u_*^3/kL = -0.12fU_g^2$ (The constant is $\mathrm{Rf}/3^{1/2}$)

Remarkably, the downward surface buoyancy flux is depends only on the geostrophic wind, and is independent of surface roughness.

The NBL structure obtained from this approach is fairly realistic. For reasonable values of u_* (0.3 m s^{-1}), U_g (10 m s^{-1}), and $f = 10^{-4}$ s^{-1}, we find that $h = 0.37(u_*L/f)^{1/2} = 150$ m, close to observed NBL depths of O(100 m). The Obhukov length $L = 56$ m, and $L/h \approx 0.38 << 1$, consistent with our original assumption that the vertical eddy mixing scale is much less than the PBL depth. The the downward surface buoyancy flux $B_0 = 1.2\times10^{-3}$ m^2s^{-3} (i. e. a virtual heat flux $(\rho c_p\theta_R/g)B_0 \approx -40$ W m^{-2}.) For $U_g = 5$ ms $^{-1}$, the downward buoyancy flux would be only 25% as large as this. These are not a large heat flux; atmospheric turbulence cannot keep the ground from cooling rapidly at night under clear skies unless the geostrophic wind is large. Instead, ground heat flux is the major counterbalance to nocturnal radiative cooling. The surface energy budgets (e. g. over a dry lake bed) nicely showed the fairly small role of surface heat fluxes in the nocturnal boundary layer.

The NBL stratification can also be deduced.

$$db/dz = N^2 = \mathrm{Ri}|d\mathbf{u}/dz|^2 = \mathrm{Ri}[u_*/L]^2|s_v|^2 = (\mathrm{Ri}/k^2\mathrm{Rf}^2)(u_*/L)^2(1 - z/h)^{-1}$$

Since Ri = Rf = 0.2, the constant is $1/k^2\mathrm{Rf} = 31$. Integrating with respect to z, we obtain

$$b(z) - b(0) = -31h(u_*/L)^2 \ln(1 - \xi)$$

This has a singularity at the BL top, which is a bit disturbing, but relates to the assumption that there must be uniform cooling all the way to the BL top, even though there is very little turbulence near the BL top. The small turbulent diffusivity then requires a large gradient there. For our example values, $N^2 = 9\times10^{-4}$ s^{-2} (2.6 K per 100 m) at the surface, rising with height. To get a more stable BL than this, we must have diabatic (e. g. radiative) cooling within the BL.

A comparison of this theory to observations is shown in the figure on the next page. It should

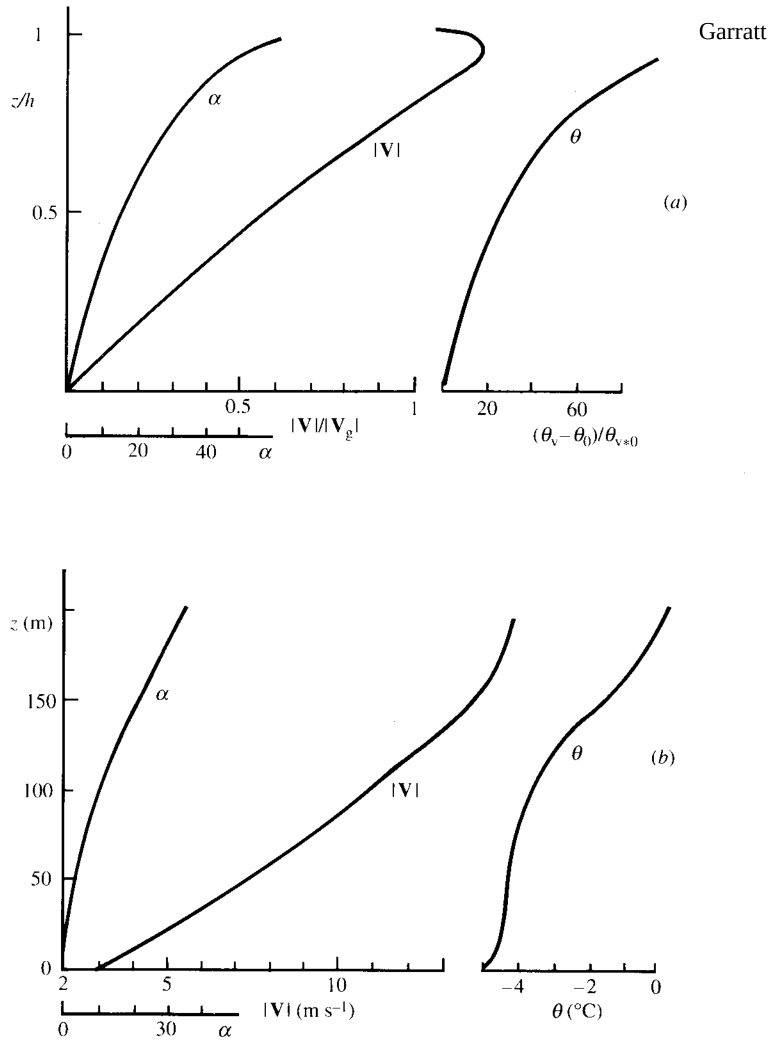


Fig. 6.15 (*a*) Predicted values of cross-isobar flow and normalized wind speed (Eq. 6.68) and of normalized temperature difference (Eq. 6.69) as functions of normalized height. (*b*) Observations from Cabaow of cross-isobar flow angle, wind speed and temperature as functions of height in the NBL. From Nieuwstadt (1985), by permission of the Oxford University Press.

Comparison of steady NBL theory (top) with tower observations (bottom) in a case of strong geostrophic wind.

be noted that this case has a high geostrophic wind speed, so that the surface buoyancy flux is large and the relative importance of radiative cooling in the NBL dynamics is smaller than usual. The comparison is quite good under these conditions. The predicted linear increase of wind with height in the BL and the concentration of the wind turning at the BL top are both observed. The observed wind turning of 30 is less than predicted, however. As predicted the strongest stratification is near the BL top. Normally, however, the NBL is most strongly stratified near the ground where clear-air radiative cooling is strongest, as seen in other soundings in these notes.

The one step in applying this approach that we have not discussed is how to relate u_* to U_g and the surface roughness z_0. The velocity profile deduced above linearly approaches zero at the surface, rather than the log-linear behavior of M-O theory. Empirical formulas, given on pp. 63-64 of Garratt, can be used to relate u_* to U_g. They are given in terms of two functions $A_2(\mu)$ and $B_2(\mu)$ of $\mu = h/L$, and are typically expressed in coordinates parallel to the surface wind. Translating these formulas into our notation, we find

$$C_g = (u_*/U_g)^2 = k^2/[(\ln(h/z_0) - A_2)^2 + B_2^2] \qquad (12)$$

where for moderately stable conditions ($0 < \mu < 35$), Garratt's eqn (3.89) implies that

$$A_2 = 1 - 0.38\mu, \ B_2 = 4.5 + 0.3\mu.$$

For our example, $C_g = (u_*/U_g)^2 = 0.0009$, and $\mu = 2.7$, so $A_2 = -0.0$, $B_2 = 5.3$. The surface roughness that could give this NBL is found by solving (13):

$$(\ln(h/z_0) - A_2)^2 + B_2^2 = k^2/C_g$$

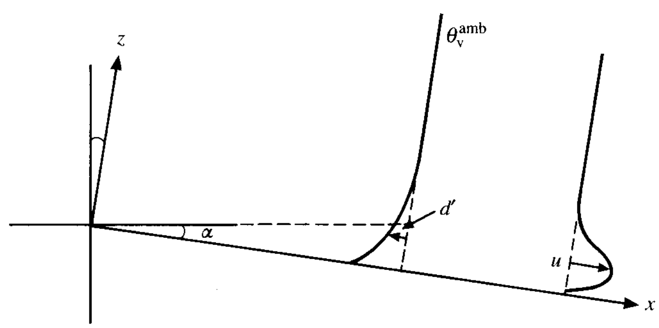
$$\Rightarrow \ln(h/z_0) = -A_2 + [k^2/C_g - B_2^2]^{1/2} = 12.2 \quad , \ z_0 \approx 0.001 \text{ m}$$

(typical of flow over a smooth land surface such as sand). A change in z_0 of several orders of magnitude is necessary to move u_* up or down by 50% for a given geostrophic wind speed.

KatabaticFlows

Sloping terrain has a large influence on stable boundary layers. The cold dense air near the surface is now accelerated by the downslope component $b \sin \alpha$ of its buoyant acceleration (α is the slope angle and $b < 0$ is the buoyancy of air within the BL relative to above-BL air at the same height). Viewed in terrain-parallel coordinates, $b \sin \alpha$ is like an effective pressure gradient force, which is strongly height-dependent since b depends on z. In this sense, the slope acts similar to a thermal wind (which would also be associated with a height dependent PGF) . Slopes of as little as 2 in 1000 can have an impact on the BL scaling.

As the slope increases, or BL stability increases, the velocity profile is increasingly determined by drag created by turbulent mixing with air above rather than surface drag. As for the NBL, the BL is typically 10s to 100s of m thick. Over glaciers, katabatic winds often occur during the day as well as during the night, since the net radiation balance of a high-albedo surface is negative even during much of the day, and evaporative cooling due to surface snowmelt can also stabilize the air near the surface. On the coast of Antarctica, persistent katabatic flows down from the interior ice-caps can produce surface winds in excess of 50 m s^{-1}.



Garratt

Fig. 6.22 Schematic representation of the downslope flow typical of night-time flow under light wind, clear sky conditions. Here, α is the slope angle and d' is the θ deficit of the flow relative to the ambient field.