Lecture 15 Cloud-topped mixed layers I

Basic dynamical equations

The simplest type of CTBL to understand is the stratocumulus-topped mixed layer, as first discussed in a classic paper by Lilly (1968, *Quart. J. Roy. Meteor. Soc.*, **94**, 292-309). The mixed layer, of depth $h(t)$, deepens through entrainment. Some types of CTBLs form preferentially under mean subsidence, so we also allow for a mean vertical motion $\overline{w}^+$ at the inversion:

$$Dh/Dt = w_e + \overline{w}^+ \quad (1)$$

It is useful to work in terms of thermodynamic variables that are approximately conserved following adiabatic fluid motions and water-phase changes. A common choice is:

Total water mixing ratio $q_t = q_v + q_l$ (water vapor + suspended liquid water)

Equivalent potential temperature $\theta_e \approx \theta \exp(Lq_v/C_pT_{LCL})$

θ_e is even approximately conserved when precipitation falls from or evaporates into the air parcel. Two other popular alternative choices of temperature variable are **moist static energy** $h = c_pT + gz + Lq_v \approx c_p\theta_e$ and **liquid water potential temperature** $\theta_l = \theta_e\exp(-Lq_t/C_pT_{LCL}) \approx \theta \exp(Lq_l/C_pT_{LCL}) \approx \theta - Lq_l/C_p$, (in most BL clouds, $q_l < 0.001$ kg kg^{-1} so $Lq_l/C_pT_{LCL} < 0.01 << 1$ so we have linearized the exponentia; we have also made the approximation that $\theta/T_{LCL} \approx 1$).

The only source terms for q_t and θ_e internal to the BL are precipitation and radiation flux convergence, respectively. The ensemble-averaged thermodynamic equations in the mixed layer are:

$$Dq_{tM}/Dt = -\partial(\overline{w'q_t'})/\partial z - \partial P/\partial z \quad (2)$$

$$D\theta_{eM}/Dt = -\partial(\overline{w'\theta_e'})/\partial z - (1/\rho_r c_p)\partial R_N/\partial z \quad (3)$$

$P(z, t) < 0$ is the vertical flux of water due to precipitation and $R_N(z, t)$ is the net upward radiative flux. Within a mixed layer, conserved thermodynamic variables and the horizontal wind velocity are constant with height at all times, so the left hand sides are height-independent. Integrating upward, and noting that the turbulent fluxes at the BL top are due to entrainment, we conclude that

$$\overline{w'q_t'} + P = (1 - z/h)(\overline{w'q_t'}_0 + P_0) + (z/h)(-w_e\Delta q) \quad (4)$$

$$\overline{w'\theta_e'} + R_N//\rho_r c_p = (1 - z/h)(\overline{w'\theta_e'}_0 + R_{N0}//\rho_r c_p) + (z/h)(-w_e\Delta\theta_e + R_N^+/\rho_r c_p) \quad (5)$$

Here the BL top at height h is treated as a discontinuous capping inversion. quantities evaluated just above the top of the BL (i. e. above all BL turbulence) have superscript $^+$, and $\Delta q_t = q_t^+ - q_{tM}$, $\Delta\theta_e = \theta_e^+ - \theta_{eM}$ are the thermodynamic jumps across the inversion. Quantities evaluated at the surface have subscript $_0$. Once the radiation and precipitation profile, the surface fluxes and the entrainment rate w_e are specified in terms of the mixed layer properties, the air-sea and the inversion jumps Δq, $\Delta\theta_e$, (4) and (5) determine the BL turbulent flux profiles and the flux divergences in (2) and (3), and we have all the information we need to integrate (1-3) forward in time

Radiation

The CTBL interacts strongly with both longwave and shortwave radiation. Hence the net upward radiation flux profile has a significant diurnal dependence, which in turn affects the BL structure. The figure above shows measures (points) and modelled (lines) up and downwelling shortwave (S) and longwave (L) fluxes in a thick summertime midlatitude stratocumulus layer at 11:30 local time, when insolation is nearly at a maximum. About 2/3 of the incident solar radiation is absorbed, but more important, about 60 W m^{-2} (6%) is absorbed in the cloud. Most of this is at more strongly absorbed near-IR wavelengths, where absorption of a photon is likely once it has scattered 10-100 times, and is concentrated toward the cloud top over a zone 100-200 m thick. The cloud droplets strongly absorb longwave radiation. Above cloud top, there is a net upward longwave flux $L_{\uparrow} - L_{\downarrow}$ of 50-100 W m^{-2}, as thermal emission from the warm cloud is larger than emission from the semitransparent and mostly colder overlying atmosphere. Within the cloud, the photon path is short and the net longwave flux is small, while below cloud base, the net flux is typically about 10 W m^{-2} due to the small difference between the SST and the cloud base temperature. The net effect (second panel) is slight radiative warming near cloud base and strong cooling in the 50 m below cloud top, with a net 60 W m^{-2} longwave cooling integrated over the BL in the case shown. Since this cooling persists over the entire diurnal cycle, it is roughly 3-4 times as large as the BL-integrated solar absorption when averaged over the diurnal cycle. Frequently, theoretical studies of cloud-topped mixed layers treat the radiative flux divergence as diurnally averaged concentrated entirely at the cloud top, and often specify it as an external parameter $\Delta R_N \approx 50$ W m^{-2} (rightmost R_N profile). In reality, of course, ΔR_N is strongly dependent on above-BL humidity, cloud and temperature, as well as cloud-top temperature and insolation. In particular, ΔR_N is largest under a clear, dry atmospheric column.

Cloud-top radiative cooling is the dominant driver for convection in most subtropical marine Sc-topped BLs. It is typically five times as large as surface buoyancy fluxes. This strong BL cooling is also what maintains the sharp 5-10 K inversions that usually top such boundary layers. The daytime solar absorption drives a process called *diurnal decoupling* in such BLs, that we will discuss later.

Precipitation

Light drizzle is sometimes observed in shallow cloud-topped boundary layers, especially when aerosol concentrations are low or the cloud is thick. A typical profile of P in a moderately drizzling Sc is shown on the next page, corresponding to cloud base precipitation of 1 mm day^{-1} and surface precipitation of 0.5 mm day^{-1}. Drizzle causes net condensation and latent heating in the cloud layer and evaporation and cooling of the subcloud layer, stabilizing the BL to convection. In a mixed layer framework, precipitation and solar absorption both tend to make the buoyancy fluxes near the cloud base of a cloud-topped mixed layer much more negative, making it difficult to sustain turbulence at those levels. Often, precipitating shallow Sc layers are observed to have some stratification of potential temperature and mixing ratio, and cloud cover may be less homogeneous.

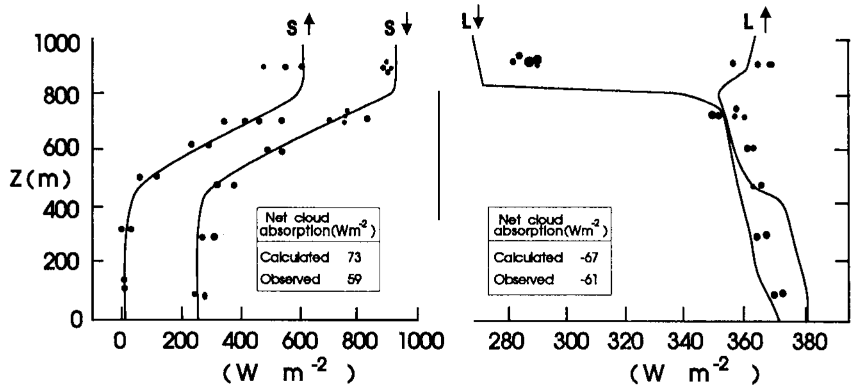
Buoyancy fluxes

To understand entrainment into a CTBL, we must first examine the buoyancy flux profile, as buoyant production is the main source of TKE in most cloud-topped mixed layers (though shear production is often also observed to be important). The buoyancy flux profile must be deduced from the fluxes of θ_e and q_t. We note that perturbation buoyancy $b' = g\theta_v'/\theta_R$. Here θ_v is defined to include the effect of liquid water loading,

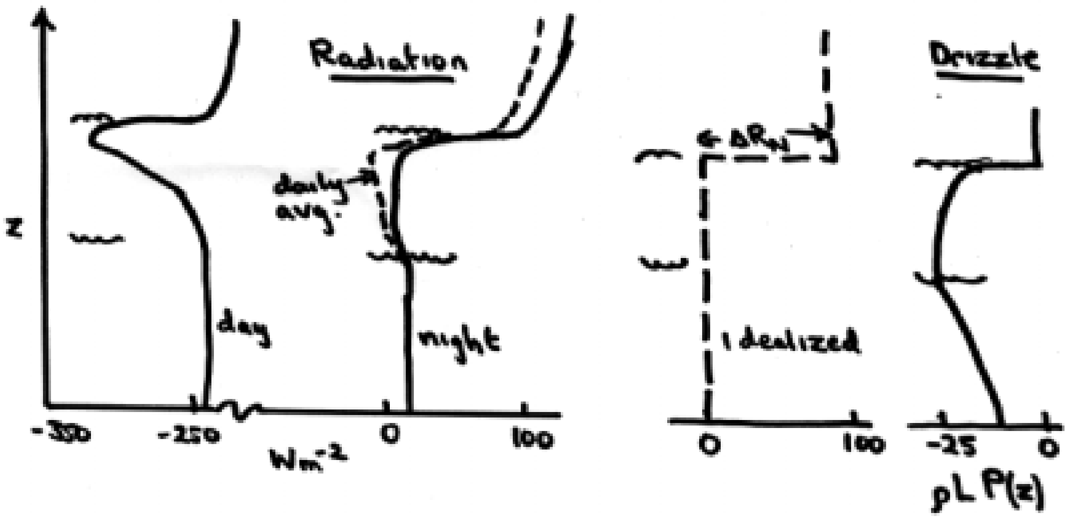
$$\theta_v = \theta(1 + \delta q_v - q_l)\ ,\ \ \delta = 0.61$$

Since q_v and q_l are small and $\theta \approx \theta_R$, we can approximate the θ_v perturbations as

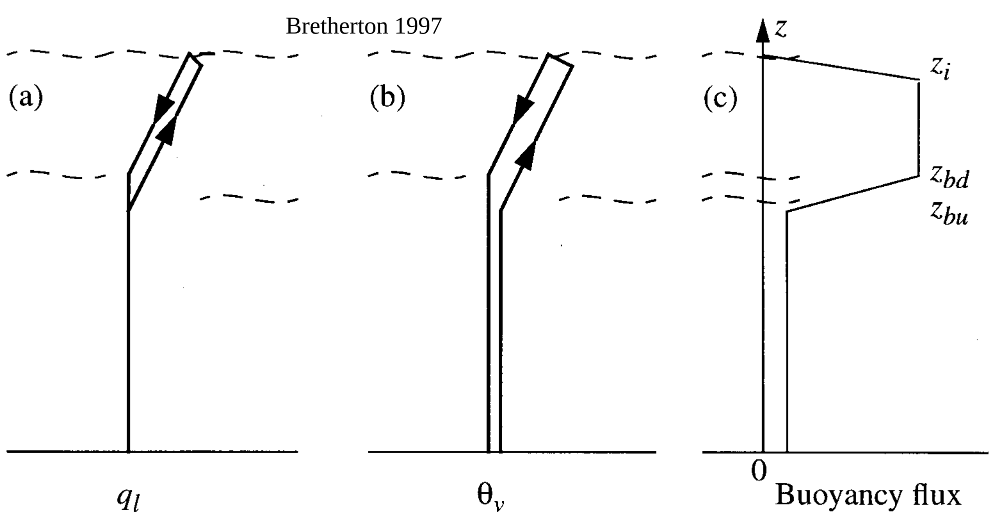
$$\theta_v' = \theta'(1 + \delta q_v - q_l) + \theta(\delta q_v' - q_l') \approx \theta' + \theta_R[\delta q_v' - q_l']$$



Radiative fluxes in North Sea stratocumulus (Nicholls 1984)



Net radiation and precipitation profiles



BL air parcel circuits

$$\approx \theta_e' - L(q_t' - q_l')/c_p + \theta_R[\delta(q_t' - q_l') - q_l']$$

$$= \theta_e' - L(1 - \delta\varepsilon)q_t'/c_p + (1 - (1 + \delta)\varepsilon)Lq_l'/c_p$$

i. e. $$\theta_v' = \theta_{vl}' + (1 - (1 + \delta)\varepsilon)Lq_l'/c_p \quad (6)$$

where

$\theta_{vl} = \theta_e - \mu L q_t/c_p$, the **liquid water virtual potential temperature**

$\varepsilon = c_p\theta_R/L \approx 0.12$

$\mu = 1 - \delta\varepsilon \approx 0.93$

In unsaturated air, $\theta_v = \theta_{vl}$. Since θ_{vl} is a linear combination of θ_e and q_t, it is also conserved by moist adiabatic processes including phase change, and its flux is simply computed from the fluxes of θ_e and q_t. From (6), we see that the buoyancy flux will jump at cloud base $z_b$by an amount

$$\overline{w'\theta_v'}(z_b^+) - \overline{w'\theta_v'}(z_b^-) = (1 - (1+\delta)\varepsilon)L\,\overline{w'q_l'}(z_b^+)/c_p \quad (7)$$

This jump can be understood by looking at an ideal air parcel circuit in the convective BL (figure on bottom of previous page) in which air moves adiabatically and without mixing from the surface (where it has been moistened) to the inversion, where it is affected by entrainment and radiative cooling. . Above a moist surface, updrafts will tend to be moister than downdrafts and will have a lower saturation level (as indicated by the two wavy dashed cloud bases in the figure), so the liquid water along the circuit will vary as in (a). This corresponds to a positive liquid water flux. In the example below, the upward moving air below cloud base is made slightly buoyant compared to the downward air by surface fluxes. Above the updraft cloud base, the upward moving air is warmed by latent heating due to condensation and follows a moist-adiabatic lapse rate. It is then cooled (mainly radiatively) at the inversion, and sinks along a moist adiabat until all liquid has evaporated. Looking at (b), we see that the updraft buoyancy is much larger above cloud base, resulting in a much increased buoyancy flux as predicted by (7). .
Below the cloud, buoyancy flux is

$$\overline{w'\theta_v'} = \overline{w'\theta_{vl}'} = \overline{w'\theta_e'} - \mu L\,\overline{w'q_t'}/c_p \text{(below cloud base)}$$

Above cloud base, one must relate liquid water to the conserved variables. Following Schubert et al. (1979, JAS, **36**, p. 1295) we linearize the Clausius-Clapeyron equation, taking $p' = 0$ and $T' = \theta'$:

$$q^{*\prime} = \gamma c_P\theta'/L, \text{ where } \gamma = (L/c_p)dq^*/dT = 1\text{-}3,$$

Hence in saturated air,

$$\theta_e' = \theta' + Lq^{*\prime}/c_p = (1 + \gamma)\theta'$$

so

$$q_l' = q_t' - q^{*\prime} = q_t' - \gamma\theta' = q_t' - (\gamma/1 + \gamma)c_P\theta_e'/L$$

Substituting this into the equation before (6), we obtain

$$\theta_v' = \theta_e' - L(1 - \delta\varepsilon)q_t'/c_p + (1 - (1 + \delta)\varepsilon)L(q_t' - (\gamma/1 + \gamma)\theta_e')/c_p$$

$$= \beta\theta_e' - \varepsilon L q_t'/c_P\ , \quad \beta = (1 + \gamma\varepsilon(1 + \delta))/(1 + \gamma) \approx 0.4\text{-}0.5.$$

Hence

$$\overline{w'\theta_v'} = \beta\overline{w'\theta_e'} - \varepsilon L\,\overline{w'q_t'}/c_p = \beta\overline{w'\theta_{vl}'} + \sigma L\,\overline{w'q_t'}/c_p \text{ (above cloud base)}$$

$$\sigma = \beta\mu - \varepsilon \approx 0.35$$

Except when the BL is strongly surface-forced, with heat fluxes of 20 W m^{-2} or more, the Bowen ratio is small, and the latent heat flux tends to be much larger than the surface virtual heat (buoyancy) flux. In this situation $\overline{w'\theta_v'} \approx 0$ below cloud base and is proportional to the latent heat flux above cloud base. Thus, the turbulence is driven primarily *from within the cloud*. Eddy motions below the cloud are due to turbulent transport as much as direct buoyancy forcing. Hence, in contrast to a surface heated BL, the turbulence tends to be strongest within the cloud, in the upper part of the BL.

Entrainment closure

There is still controversy over the best choice of entrainment closure. Two classes of approach have been widely used, called **flux-partitioning** and ***w*∗ or eddy-velocity** closures by Bretherton and Wyant (1997, JAS, **54**, 148-167), from which much of the development below is taken. Both in their simplest form assume that the BL is convectively driven, i. e. that shear generation of TKE is negligible above the surface layer, though generalizations applicable when shear is important exist.

In one form of flux partitioning, *geometric partitioning*, it is assumed that the vertical integral of the buoyancy flux over the height range at which it is negative is $-I_N$, the vertical integral of the buoyancy flux over the height range at which it is positive is I_P, and the entrainment rate is deduced by requiring that

$I_N = -cI_P$, (c = 0.04 is an empirically determined constant) - flux-partitioning approach (8)

The given value of c is consistent with a surface-heated dry BL with buoyancy flux B_0 at the bottom, decreasing to $-0.2B_0$ at the top. Since the buoyancy fluxes (and hence I_N and I_P) depend on w_e, (6) is an implicit equation for w_e. Under most reasonable conditions it has a unique solution for given surface fluxes, radiation and drizzle. Many slight variations of (8) have also been used, with little fundamental effect on the mixed layer dynamics. A fundamental issue in applying (8) to a CTBL is the radically different structure of the buoyancy flux profile compared to a dry BL. Because of the strong radiative and evaporative cooling near the cloud top, downdraft air tends to be negatively buoyant within a few tens of meters of the inversion, despite the entrainment of warmer air. Hence the inversion zone makes only a very small contribution to I_N. Hence, (8) is balanced in a cloud-topped mixed layer model by negative buoyancy fluxes below the cloud base (a region far from the entrainment zone). This contradicts the original philosophy of closures such as (8), namely that the penetrative nature of convection (negative buoyancy fluxes) drives entrainment.

Another form of flux partitioning called *process partitioning* (Stage and Businger 1981; Lewellen and Lewellen 1998) divides the buoyancy flux integral into that part I_E associated with entrainment fluxes of q_t and θ_e, and the remaining 'nonentrainment' part I_{NE} with all other processes. The closure is to assume that $I_E = c'I_{NE}$. The choice $c' = 0.2$ would fit a dry convective boundary layer, but for a cloud-topped boundary layer $c' \approx$ 0.35-0.4 fits LES simulations reasonably. The physical mechanism that would enforce such a closure is unclear.

An eddy-velocity closure postulates that w_e is determined by some measures of the large-eddy velocity and the inversion strength. Lab experiments (Turner 1968) based on stirred-grid turbulence beneath strong density jumps, the dry surface-heated BL, and lab analogues to a BL radiatively cooled from its top all are consistent with the relation

$$w_e = Aw_*/\mathrm{Ri} \quad = \quad Aw_*^3/\, h\Delta b, \tag{9}$$

$w_*^3 = 2.5 \int_0^{z_i} dz\ \overline{w'b'}(z)$ is convective velocity scale, typically 0.5-1 m s^{-1} for CTBLs

$\mathrm{Ri} = h\Delta b/w_*^3$ is nondim. inversion strength, typically 30-300 for CTBLs

$A \approx 0.2$ for dry and lab convection, 2-5 for aircraft-observed CTBLs

It has proven very difficult to directly measure w_e in CTBLs, which is often less than 1 cm s^{-1}, to accuracies of better than 50%. However, the above range of cloud values of A, from an influential paper by Nicholls and Turton (1986), clearly suggests entrainment is much more efficient in clouds than in a clear BL for a given large eddy scale and inversion strength, which is a bit puzzling. Nicholls and Turton (1986) proposed that this was due to evaporative cooling of mixtures of entrained unsaturated air and cloudy air, which allows them to sink more readily into the cloud than mixing were not accompanied by evaporative cooling, and suggested that A depends strongly on a measure of the potential for cloud-top evaporative cooling by mixing. However, this is still a matter of considerable debate. Nicholls and Turton showed that their closure fit their data much better than flux-partitioning closures, but uncertainties were too large to completely rule out flux-partitioning closures. Large-eddy simulations have also been used to predict the dependence of w_e on relevant parameters, but because of the strong inversions that typify cloud-topped mixed layers, it is still unclear whether we have yet achieved a high enough resolution within the entrainment zone at the inversion to confidently simulate w_e. This is a currently very actively researched area, and a variety of entrainment closures have recently been proposed that do not fit into either of the above categories.

Energy balance and entrainment

In marine CTBLs, where surface buoyancy fluxes often tend to be small, the entrainment rate is largely dictated by the energy balance of the BL. Use of a different entrainment closure for predicting BL evolution has surprisingly little impact on the BL depth, because although it can change the *relative* magnitude of the subcloud buoyancy fluxes considerably, it will generally select a w_e for which the subcloud buoyancy fluxes are small in an absolute sense, i. e. compared to the net radiative cooling.

With this in mind, let us consider the energy (or more precisely θ_{vl}) balance of a mixed layer in which the surface buoyancy (i. e. θ_{vl}) flux is approximately zero. Note that this implies that the mixed layer $\theta_{vlM} \approx \theta_{v0}$, the surface value. This can be obtained from (2) and (3):

$$\rho c_p h\, d\theta_{vlM}/dt \;\approx\; \rho c_p w_e(\theta_v^+ - \theta_{vlM}) \;-\; \{R_N^+ - R_N(0)\} \;-\; \mu\rho LP(0)$$

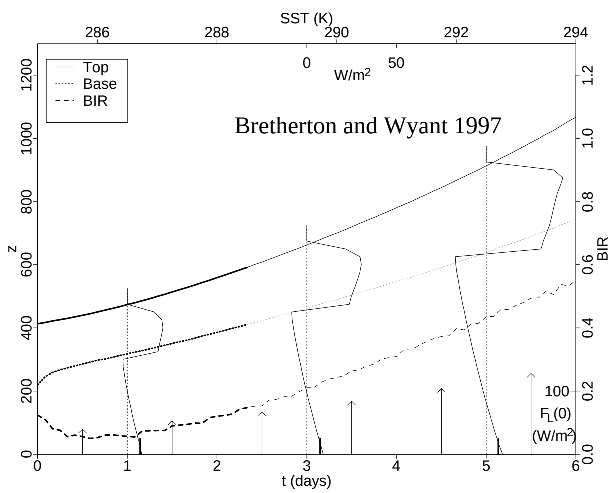
Storage Entrainment Radiation Latent Heating

For a 500 m BL warming at 2 K day^{-1}, the left hand side would be 10 W m^{-2}. Usually, the latent heating term is also 10 W m^{-2} or less. The radiation term is -50 W m^{-2}, so to a large extent, the entrainment term must balance it:

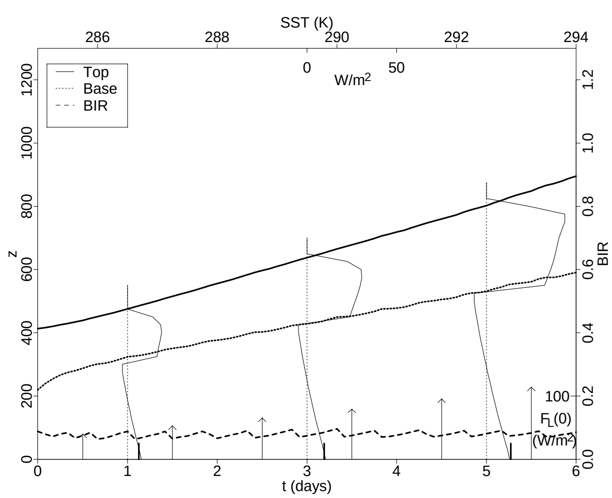
$$\rho c_p w_e(\theta_v^+ - \theta_{vlM}) \approx R_N^+ - R_N(0)$$

In the surface heated BL, the rate of deepening depended strongly on the surface buoyancy flux and ambient stratification, but only fairly weakly on the entrainment closure (e. g., think about the encroachment solution, which isn't all that different from if realistic entrainment is modelled). A similar result holds for the CTBL -- *as long as the BL remains well-mixed the BL evolution is insensitive to the entrainment closure used*.

The figure below shows parallel runs of a cloud-topped mixed layer model with an eddy-strength and a flux-partitioning entrainment closure. The simulations assume realistic above atmospheric temperature and moisture profiles, a fixed 50 W m^{-2} BL radiative flux divergence, and an SST warming at 1.5 K day^{-1}, corresponding to an air column moving over a warmer ocean surface. The buoyancy flux profiles for the two models both show strong upward buoyancy fluxes within the cloud, but differ considerably below cloud base, to satisfy the two different entrainment closures. In fact, the eddy-strength closure predicts an increasingly strong and deep layer of negatively buoyancy fluxes incompatible with the continued maintenance of mixed layer structure. However, both models show quite similar evolutions of cloud top and cloud base, showing the insensitivity of BL evolution to entrainment closure.



Mixed layer evolution with w_* closure with $A = 2$



Mixed layer evolution with flux-partitioning closure