Lecture 5. Surface roughness and the logarithmic sublayer

(Garratt, Ch 3: similarity theory; Ch. 4: surface characteristics)

Near a solid boundary, in the 'surface layer', vertical fluxes are transported primarily by eddies with a lengthscale much smaller than in the center of the BL. A very successful similarity theory is based on dimensional reasoning (Monin and Obuhkov, 1954). It postulates that near any given surface, the wind and thermodynamic profiles should be determined purely by the height z above the surface (which scales the eddy size) and the surface fluxes which drive turbulence:

1. Surface mom. flux $\overline{u'w'}_0$ (often expressed as **friction velocity** $u_* = (\overline{u'w'}_0)^{1/2}$)
2. Surface buoyancy flux $B_0 = \overline{w'b'}_0$

One can construct from these fluxes the

Obuhkov length $L = -u_*^3/kB_0$ (positive for stable, negative for unstable BLs)

Here $k = 0.4$ is the **von Karman constant**, whose physical significance we'll discuss shortly. In the ABL, a typical u_* might be 0.3 m s^{-1} and a typical range of buoyancy flux would be -3×10^{-4} m^{2}s^{-3}(nighttime) to 1.5×10^{-2} m^{2}s^{-3}(midday) (i. e. a virtual heat flux of -10 W m^{-2} at night, 500 W m^{-2} at midday), giving L = 200 m (nighttime) and -5 m (midday).

The logarithmic sublayer (Garratt, p. 41)

At height z, the characteristic eddy size, velocity, and buoyancy scale with z, u_*, and B_0/u_*. If the buoyant acceleration acts over the eddy height, it would make a vertical velocity $(z\delta b)^{1/2} = (zB_0/u_*)^{1/2}$. If $z < |L|$, this buoyancy driven contribution to the vertical velocity is much smaller than the shear-driven inertial velocity scale u_*, so buoyancy will not significantly affect the eddies. In this case, the mean wind shear will depend only on u_* and z, so dimensionally

$$d\bar{u}/dz = u_*/kz \quad (z < |L|) \tag{1}$$

This can also be viewed in terms of mixing length theory, with eddy diffusion

$$K_m \propto \text{(velocity)(length)} = (u_*)(kz)$$

$$\overline{u'w'}_0 = -K_m d\bar{u}/dz \quad \Rightarrow \quad u_*^2 = ku_*z\, d\bar{u}/dz \text{ (equivalent to (1))}$$

The von Karman constant k is the empirically determined constant of proportionality in (1). Integrating, we get the **logarithmic velocity profile law**:

$$\bar{u}(z)/u_* = k^{-1}\ln(z/z_0) \quad (z << |L|) \tag{2}$$

The constant of integration z_0 depends on the surface and is called the **roughness length**. It is loosely related to the typical height of closely spaced surface obstacles, often called roughness elements (e. g. water waves, trees, buildings, blades of grass). It depends on the distribution as well as the height h_c of roughness elements (see figure below), but as a rule of thumb,

$$z_0 \sim 0.1h_c$$

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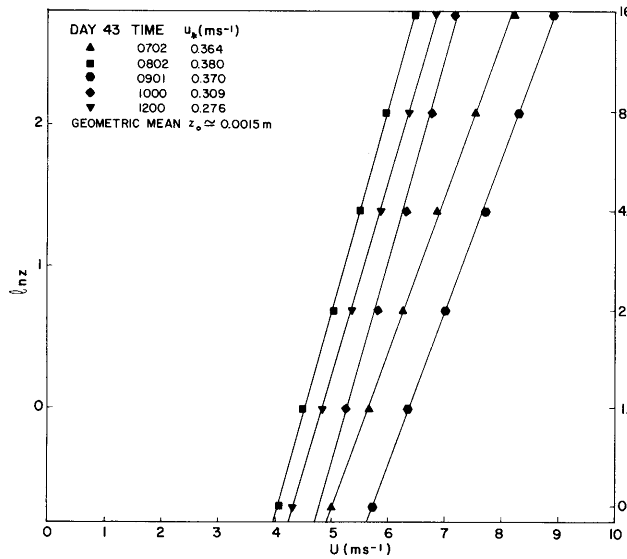

Fig. 10.4 Comparison of the observed wind profiles in the neutral surface layer of day 43 of the Wangara Experiment with the log law [Eq. (10.6)] (solid lines). [Data from Clarke *et al.* (1971).]

Example of logarithmic velocity profile in a neutral surface layer.

Garratt

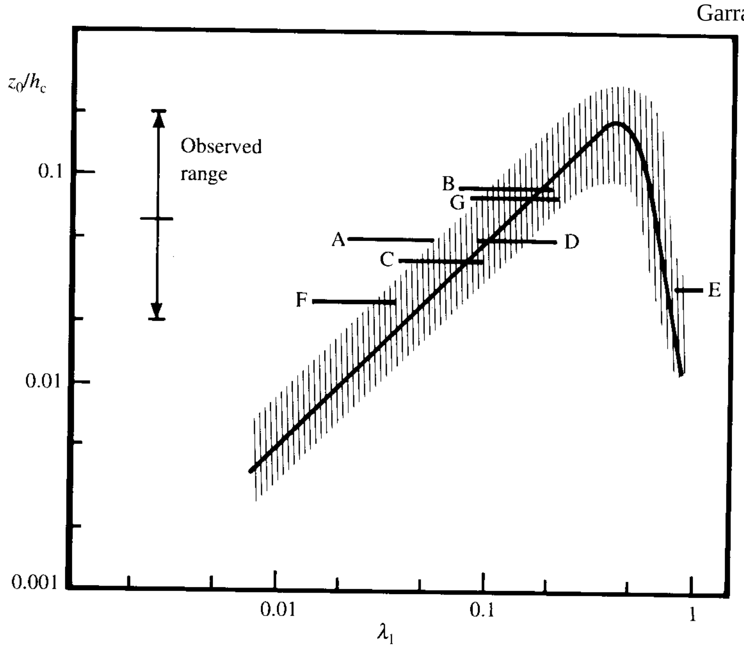

Fig. 4.1 Variation of z_0/h_c with element density, based on the results of Kutzbach (1961), Lettau (1969) and Wooding *et al.* (1973), represented by the shaded area and solid curve. Some specific atmospheric data are also shown as follows: A and B, trees; C and D, wheat; E, pine forest; F, parallel flow in a vineyard; G, normal flow in a vineyard. Analogous wind-tunnel data are described in Seginer (1974). From Garratt (1977b).

Dependence of roughness length on density λ of roughness elements.

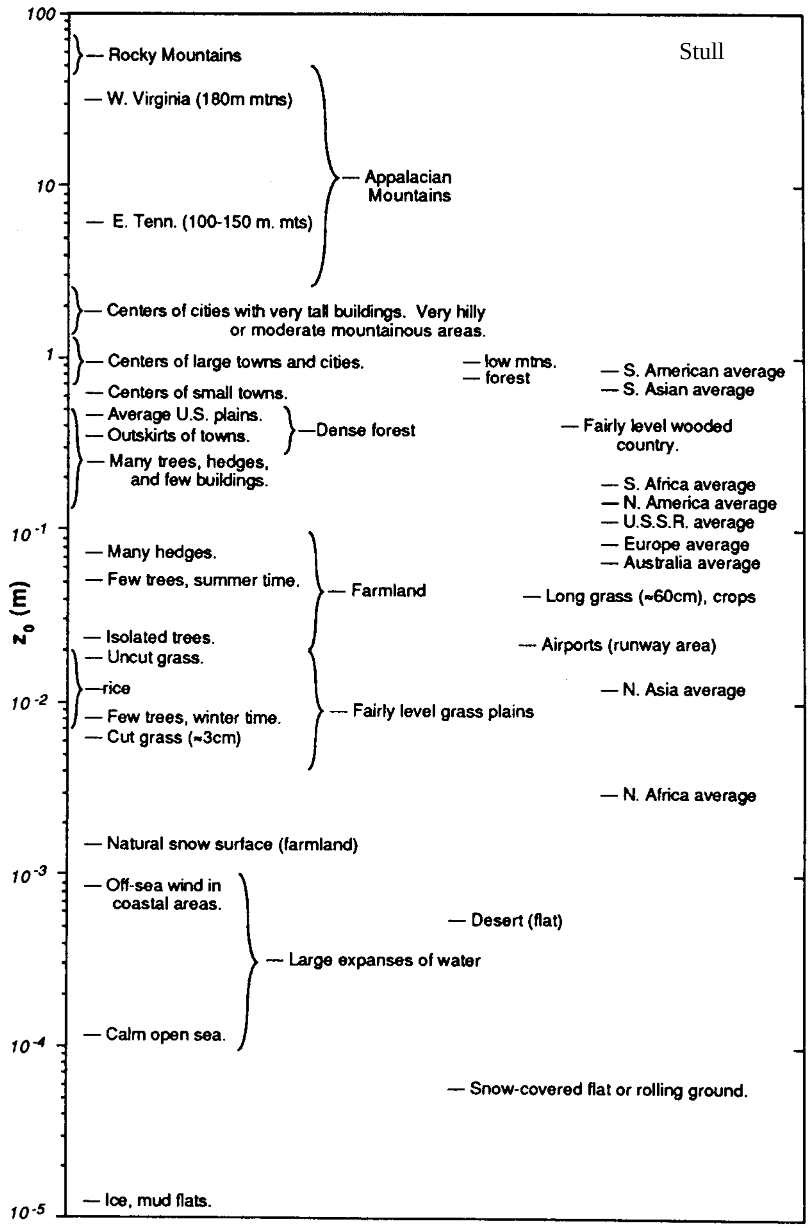

Fig. 9.6 Aerodynamic roughness lengths for typical terrain types. (After Garratt 1977, Smedman-Högström & Högström 1978, Kondo & Yamazawa 1986, Thompson 1978, Napo 1977, and Hicks 1975).

z_0 varies greatly depending on the surface, but a typical overall value for land surfaces is z_0 = 0.1 m (see table on next page). In the rare circumstance that the surface is so smooth that the viscous sublayer is deeper than roughness elements,

$$z_0 \sim 0.1\nu/u_* \sim 0.015 \text{ mm for } u_* = 0.1 \text{ m s}^{-1}$$

Near the surface, the log profile fits best if z is offset by a **zero-plane displacement** d_0 which lies between 0 and the height h_c of roughness elements, and is typically roughly $0.7h_c$.

$$\overline{u}(z)/u_* = k^{-1} \ln([z-d_0]/z_0) \quad (z << |L|) \tag{3}$$

Roughness of Water Surfaces (Garratt, p. 97-100)

The roughness of a water surface depends on wind speed and the spectrum of waves. A strong wind blowing from S to N across the SR 520 bridge shows the importance of fetch on wave spectrum. On the south side, large waves will be crashing onto the bridge deck. On the N side, the water surface will be nearly smooth except for short wavelength ripples ('cats paws') associated with wind gusts. As one looks further N from the bridge, one sees chop, then further downwind, longer waves begin to build. It can take a fetch of 100 km for the wave spectrum to reach the steady state or fully developed sea assumed by most formulas for surface roughness. It is thought that much of the wind stress is associated with boundary layer separation at sharp wave crests of breaking waves or whitecaps, which start forming at wind speeds of 5 m s^{-1} and cover most of the ocean surface at wind speeds of 15 m s^{-1} or more.

For wind speeds below 2.5 m s^{-1}, the water surface is approximately aerodynamically smooth, and the viscous formula for z_0 applies. For intermediate wind speeds, the flow is aerodynamically smooth over some parts of the water surface but rough around and in the lee of the breaking whitecaps, and for wind speeds above 10 m s^{-1} it is fully rough. For rough flow, Charnock (1955) suggested that z_0 should depend only on the surface stress on the ocean and the gravitational restoring force, i. e., u_* and g, leading to **Charnock's formula**:

$$z_0 = \alpha_c u_*^2/g, \quad (\alpha_c = 0.016 \quad 20\% \text{ from empirical measurements}).$$

This formula appears reasonably accurate for 10 m wind speeds of 4-50 m s^{-1}. For 10 m wind speeds of 5-10 m s^{-1}, this gives roughness lengths of 0.1 - 1 mm, much less than almost any land surface. Even the heavy seas under in a tropical storm have a roughness length less than mown grass! This is because (a) the large waves move along with the wind, and (b) drag seems to mainly be due to the vertical displacements involved directly in breaking, rather than by the much larger amplitude long swell. The result is that near-surface wind speeds tend to be much higher over the ocean, while surface drag tends to be smaller over the ocean than over land surfaces.

Snow and Sand Surfaces (Garratt, p. 87-88)

The roughness of sand or snow surfaces also increases of wind speed, apparently due to suspension of increasing numbers of particles. Charnock's dimensional argument again applies, and remarkably, the same α_c appears to work well, though now the minimum z_0 is larger (typically at least 0.05 mm), associated with the roughness of the underlying solid surface.

Bulk Aerodynamic Drag Formula (Garratt, p. 100-101)

Suppose that a wind measurement is taken at a standard reference level z_R within the log layer (A typical shipboard height of z_R = 10 m is often used for ocean measurements). Then (ignoring zero-plane displacement for simplicity), $\overline{u}(z_R) = u_* k^{-1} \ln(z_R/z_0)$. The bulk aerodynamic formula relates the surface stress $\rho_0\overline{u'w'}$ to the reference wind speed in terms of a drag coefficient C_{DN} which depends on surface roughness:

$$-\rho_0\overline{u'w'} = \rho_0 u_*^2 = \rho_0 C_{DN}\overline{u}^2(z_R),$$

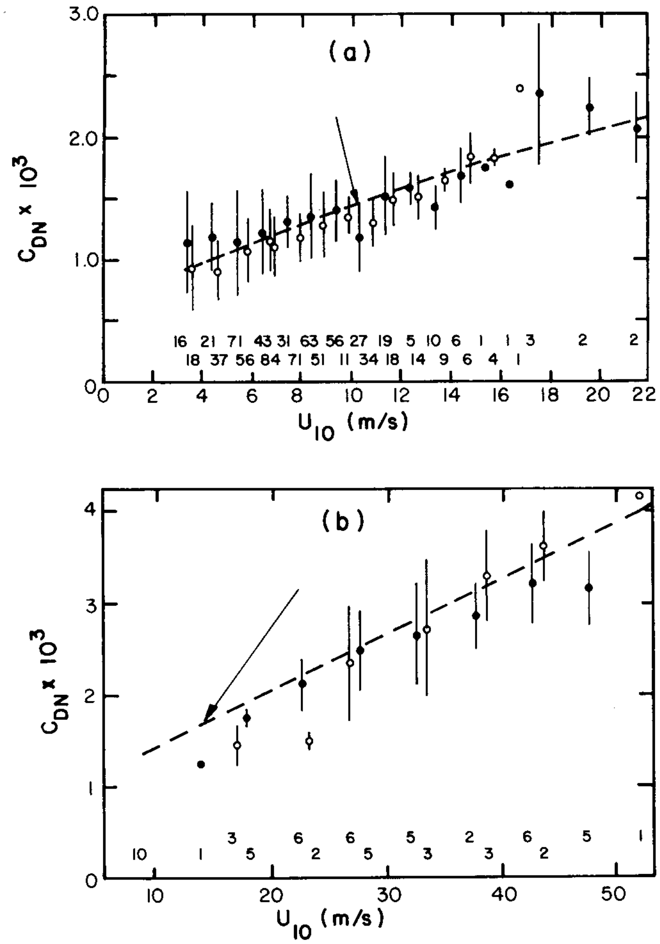

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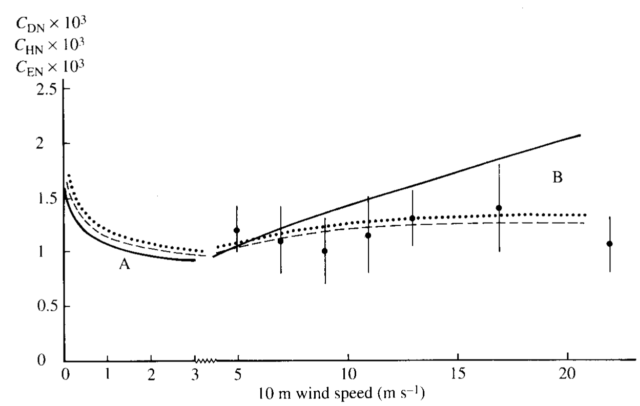
Fig. 13.4 Neutral drag coefficient as a function of wind speed at a 10-m height compared with Charnock's formula [Eq. (13.5), indicated by the arrows in (a) and (b)] with $a = 0.0144$. Block-averaged values are shown for (a) 1-m sec^{-1} intervals, based on eddy correlation and profile methods, and (b) 5-m sec^{-1} intervals, based on geostrophic departure method and wind flume simulation experiments. [After Garratt (1977).]

(4)



Garratt

Fig. 4.9 Drag coefficient C_{DN}, heat transfer coefficient C_{HN} and water vapour transfer coefficient C_{EN} as functions of the 10 m wind speed. Curves A are for smooth flow: solid curve C_{DN} (Eq. 4.22); pecked curve, C_{HN} (Eqs. 4.10 and 4.26a); dotted curve, C_{EN} (Eqs. 4.11 and 4.26b). Curves B are for rough flow: solid curve, C_{DN} (Eq. 4.23); pecked curve, C_{HN} (Eqs. 4.10 and 4.27); dotted curve, C_{EN} (Eqs. 4.11 and 4.28). Observational data are from Large and Pond (1982).

$$C_{DN} = k^2/\{\ln(z_R/z_0)\}^2 \tag{5}$$

The *N*, for 'neutral', in the suffix is to remind us that this formula only applies if when $z_R << |L|$, which for typical reference heights (2 m or 10 m) requires fairly neutrally stratified conditions, as are often observed over the oceans but less often over land. For z_R = 10 m wind speed and z_0 = 0.1 m, $C_{DN} = 8 \times 10^{-3}$

Over the water, C_{DN} is a function of surface roughness u_* and hence implicitly of wind speed. While Charnock's formula gives an awkward transcendental equation to solve for C_{DN} in terms of $\overline{u}(z_R)$, a good approximation using mean 10 m wind speed u_{10} is:

$$C_{DN} = (0.75 + 0.067u_{10}) \times 10^{-3} \quad \text{(water, neutrally stratified BL)}$$

Heat and Moisture Transfer in Neutral Conditions

Let *a* be a scalar (θ, *q*, etc.) transported by the turbulence. In the log-layer, we again might hope for a flux-gradient relation of the form

$$\overline{w'a'} = K_a d\overline{a}/dz, \quad K_a = k_a z u_*$$

The nondimensional constant k_a need not equal the von Karman constant, since momentum perturbations of fluid parcels are affected by eddy-induced pressure gradients, while scalars are not. However, empirical measurements do suggest that $k_a = k$ in a neutral BL. A scale for turbulent perturbations a' in the log layer is:

$$a_* = \overline{w'a'}|_0/u_*$$

Since the flux is approximately equal to its surface value throughout the surface layer,

$$d\overline{a}/dz = -\overline{w'a'}|_0 / (kzu_*) = -a_*/kz$$

$$\overline{a}(z) - \overline{a}_0 = -a_*/k \ln(z/z_{0a})$$

This has the same logarithmic form as the velocity profile, but the scaling length z_{0a} need not be (and usually isn't) the same as z_0. In fact, it is often much smaller, because pressure (form) drag on roughness elements helps transfer momentum between the interfacial (viscous) sublayer around roughness elements to the inertial sublayer . No corresponding nonadvective transfer mechanism exists for scalars, so they will be transferred less efficiently out of the interfacial layer ($z_a < z_0$) unless their molecular diffusivity is much larger than that of heat.

This can be converted into a bulk aerodynamic formula like (5), but the transfer coefficient may be different:

$$\rho_0\overline{w'a'} = \rho_0 C_{aN}\overline{u}(z_R)\{\overline{a}_0 - \overline{a}(z_R)\},$$

$$C_{aN} = k^2/\{\ln(z_R/z_0)\ln(z_R/z_{0a})\}$$

For most land surfaces, the heat and moisture scaling lengths z_{0H} and z_{0q} are 10-30% as large as z_0, resulting in typical C_{HN} of 0.7-0.95 C_{DN} . For water surfaces, the heat and moisture coefficients are comparable to C_{DN} for 10 m winds of 7 m s^{-1} or less, but remain around 1-1.5 $\times 10^{-3}$ rather than increasing as wind speed increases. This corresponds to heat and moisture scaling lengths appropriate for laminar flow even at high wind speeds. For instance, ECMWF uses z_{0H}, z_{0q} = (0.4, 0.62)ν/u_* following Brutsaert (1982).

Bulk aerodynamic formulas are quite accurate as long as (i) an appropriate transfer coefficient

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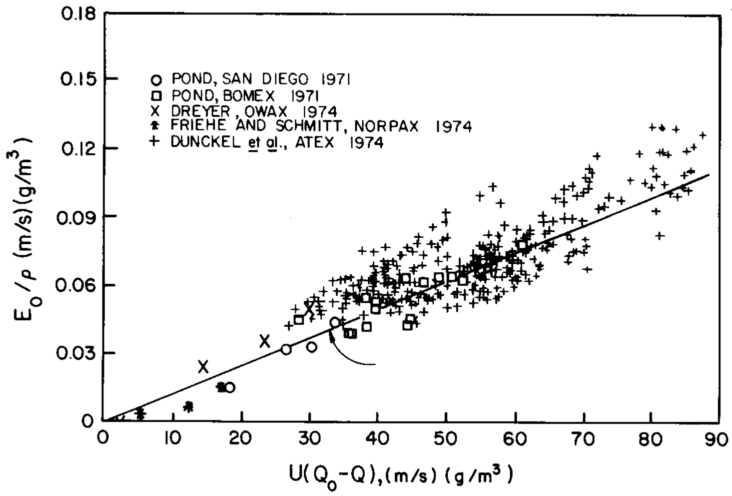

Fig. 13.6 Observed moisture flux at the sea surface as a function of $U(Q_0 - Q)$ compared with Eq. (13.8) with $C_W = 1.32 \times 10^{-3}$, indicated by the arrow. [After Friehe and Schmitt (1976).]

is used for the advected quantity, the reference height, and the BL stability, and (ii) Temporal variability of the mean wind speed or air-sea differences are adequately sampled. The figure below shows comparisons between direct (eddy-correlation) measurements of moisture flux in nearly neutrally stratified BLs over ocean surfaces compared with a bulk formula with constant $C_q = 1.32\times10^{-3}$. In individual cases, discrepancies of up to 50% are seen (which are as likely due to sampling scatter in the measured fluxes as to actual problems with the bulk formula), but the overall trend is well captured. However, due to this type of scatter, no two books or papers seem to exactly agree on the appropriate formulas to use, though all agree within about 20%.