Lecture 12

Review for Exam 1: IR radiative processes

Topics to review:

- **Basic radiometric quantities: intensity and flux.**

Relation between flux and intensity: $F_l = \int_\Omega I_l \cos(q) d\Omega$ [2.6]

For isotropic atmosphere: $F = p\ I$

- **The Beer-Bouguer-Lambert law.**

For extinction: $dI_l = -b_{e,l} I_l ds$

For emission: $dI_l = b_{e,l} J_l ds$

Integral form of the extinction law:

$$I_l = I_0 \exp(-t)$$ [2.7]

- **Concepts of extinction (scattering + absorption) and emission. Optical depth.**

$$t_l(s_1; s_2) = \int_{s_1}^{s_2} b_{e,l} ds = \int_{s_1}^{s_2} r b^*_{e,l} ds = \int_{s_1}^{s_2} N s_{e,l} ds$$ [2.8]

Volume extinction coefficient (LENGTH^{-1})

Mass extinction coefficient (LENGTH^{2} MASS^{-1})

Extinction cross section (LENGTH^{2})

- **Concepts of a blackbody, thermodynamical equilibrium, and local thermodynamical equilibrium**.

- **Main radiation laws:**
  - Planck function
  - Stefan-Boltzmann law: $\mathbf{F = p\ B(T) = s_b\ T^4}$
  - Wien's displacement law
  - Kirchhoff's law

- **The general equation of radiative transfer:**

$$dI_l = -b_{e,l} I_l ds + b_{e,l} J_l ds$$

$$\frac{dI_l}{b_{e,l} ds} = -I_l + J_l \qquad [3.4]$$

$$-\frac{dI_l}{dt_l} = -I_l + J_l \qquad [3.5]$$

$$\frac{dI_l}{dt_l} = I_l - J_l$$

In the approximation of the plane-parallel atmosphere:

$$m\frac{dI_l(t;m;j)}{dt} = I_l(t;m;j) - J_l(t;m;j) \qquad [3.8]$$

- **Absorption by atmospheric gases. Absorption coefficient and transmission function.**

<u>Spectral line shapes</u>:

- Lorentz profile (**pressure broadening**)

$$f_L(\nu - \nu_0) = \frac{\alpha}{\pi\left[(\nu - \nu_0)^2 + \alpha^2\right]} \qquad [5.1]$$

- Doppler profile

$$f_D(\nu - \nu_0) = \frac{1}{\alpha_D\sqrt{\pi}} \exp\left[-\left(\frac{\nu - \nu_0}{\alpha_D}\right)^2\right] \qquad [5.2]$$

$$\alpha_D = \frac{\nu_0}{c}(2k_B T/m)^{1/2}$$

- Voigt profile

<u>Absorption coefficient</u>:

$$k_\nu = S\, f(\nu - \nu_0) \qquad [5.4]$$

<u>Transmission:</u>

$$T_\nu = \exp(-\tau_\nu) = \exp\left(-\int_u k_\nu\, du\right)$$

- **Spectral transmission and absorption:** **(see Lecture 7)**

$$T_{\bar{\nu}}(u) = \frac{1}{\Delta\nu}\int_{\Delta\nu} \exp(-k_\nu u)\, d\nu = \frac{1}{\Delta\nu}\int_{\Delta\nu} \exp(-S f(\nu - \nu_0) u)\, d\nu$$

$$A_{\bar{\nu}}(u) = 1 - T_{\bar{\nu}}(u)$$

Approximation of weak line absorption

$$A_{\bar{n}}(u) = \frac{Su}{\Delta n}$$

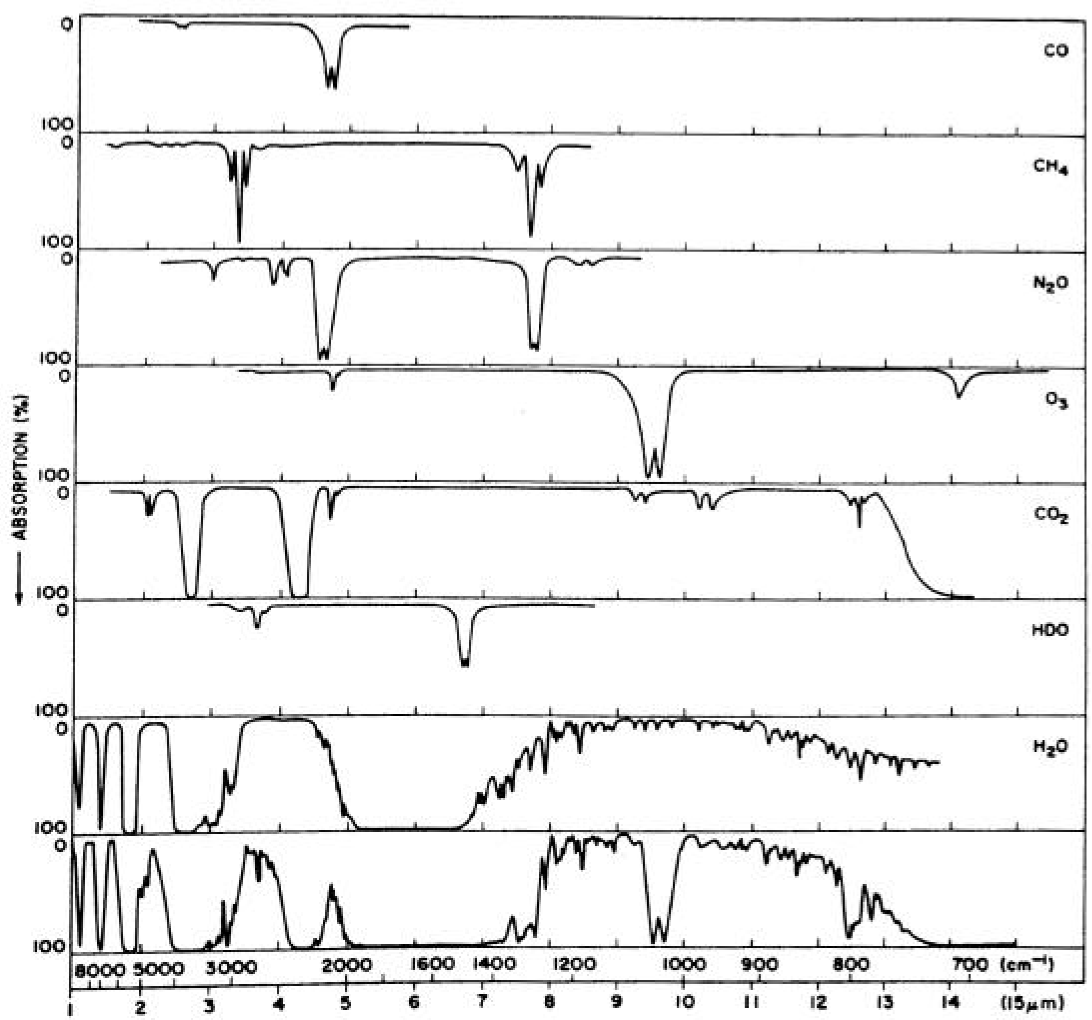
Approximation of strong line absorption

$$A_{\bar{n}}(u) = 2\frac{\sqrt{Su\,a}}{\Delta n}$$

- **IR absorption spectra of main atmospheric gases (H_2O, CO_2, O_3, CH_4, N_2O, CFCs).**

(see Figure 5.5 and Table 5.3 in Lecture 5)

Figure 5.5 Low-resolution infrared absorption spectra of the major atmospheric gases.



- **IR radiative transfer in the plane-parallel atmosphere:**

$$m\frac{dI_n^{\uparrow}(t;m)}{dt} = I_n^{\uparrow}(t;m) - B_n(T) \qquad [6.1a]$$

$$-m\frac{dI_n^{\downarrow}(t;-m)}{dt} = I_n^{\downarrow}(t;-m) - B_n(T) \qquad [6.1b]$$

<u>Solutions:</u>

$$I_n^{\uparrow}(t;m) = I_n^{\uparrow}(t_1;m)\exp\left(-\frac{t_1 - t}{m}\right)$$

$$+ \frac{1}{m}\int_t^{t_1} \exp\left(-\frac{t' - t}{m}\right) B_n(T(t'))\,dt' \qquad [6.2a]$$

$$I_n^{\downarrow}(t;-m) = I_n^{\downarrow}(t_1;m)\exp\left(-\frac{t}{m}\right)$$

$$+ \frac{1}{m}\int_0^{t} \exp\left(-\frac{t - t'}{m}\right) B_n(T(t'))\,dt' \qquad [6.2b]$$

Solution expressed via transmission (assuming that surface is a blackbody and there is no source of the IR radiation at the top of the atmosphere):

$$I_n^{\uparrow}(t;m) = B_n(t^*)T_n((t^* - t)/m)$$

$$-\int_t^{t^*} B_n(t')\frac{dT_n((t'-t)/m)}{dt'}dt' \qquad [6.4a]$$

$$I_n^{\downarrow}(t;-m) = \int_0^t B_n(t')\frac{dT_n((t-t')/m)}{dt'}dt' \qquad [6.4b]$$

<u>Fluxes:</u>

$$F_n^{\uparrow} = 2p\int_0^1 I_n^{\uparrow}(m)m\,dm \;;\; F_n^{\downarrow} = 2p\int_0^1 I_n^{\downarrow}(-m)m\,dm$$

<u>Solution for fluxes</u>

$$F_n^{\uparrow}(t) = 2pB_n(t^*)\int_0^1 \exp(-\frac{t^*-t}{m})m\,dm$$

$$+\,2p\int_0^1\int_t^{t^*}\exp(-\frac{t'-t}{m})\;B_n(t')dt'dm \qquad [8.2a]$$

$$F_n^{\downarrow}(t) = 2p\int_0^1\int_0^t \exp(-\frac{t-t'}{m})\;B_n(t'))dt'dm \qquad [8.2b]$$

Monochromatic diffuse transmission function (or transmittance):

$$T_n^f(t) = 2\int_0^1 T_n(t/m)\,m\,dm \qquad [8.5]$$

<u>Solutions for total fluxes using diffuse (slab) transmission:</u>

$$F^{\uparrow}(u) = \int_0^{\infty} pB_n(T_s)T_n^f(u)\,dn$$

$$+\int_0^{\infty}\int_0^{u} p\,B_n(T(u'))\frac{dT_n^f(u-u')}{du'}\,du'dn \qquad [8.10a]$$

$$F^{\downarrow}(u) = \int_0^{\infty}\int_{u^*}^{u} pB_n(T(u'))\frac{dT_n^f(u'-u)}{du'}\,du'dn \qquad [8.10b]$$

- **Infrared radiative heating/cooling rates.**

$$\left(\frac{\partial T}{\partial t}\right)_{IR} = -\frac{1}{c_p r}\frac{dF(z)}{dz} \qquad [8.8]$$

where the **net flux divergence** for the layer **Dz** is

$$\Delta F = F(z+\Delta z) - F(z)$$

and net flux is

$$F(z) = F^{\uparrow}(z) - F^{\downarrow}(z)$$

(see discussion of IR fluxes and cooling rates in Lecture 11)