

## Lecture 19

### Methods for solving the radiative transfer equation. Part 2: Effects of surface reflection on the atmospheric radiation field.

#### Objectives:

1. Surface reflection.
2. Inclusion of the surface reflection into the radiative transfer.

#### Required reading:

L80: 6.5;

### 1. Surface reflection.

Surfaces can modify the atmospheric radiation field by

- a) reflecting a portion of the incident radiation back into the atmosphere;
- b) emitting the thermal radiation (see Lecture 3, Kirchhoff's law);
- c) absorbing a portion of incident radiation ((see Lecture 3, Kirchhoff's law);
- d) transmitting some incident radiation.

*Two extreme types of the surface reflection:*

**specular reflectance** and **diffuse reflectance**.

**Specular reflectance** is the reflectance from a perfectly smooth surface (e.g., a mirror):

Angle of incidence = Angle of reflectance

- Reflection is generally **specular** when the "roughness" of the surface is smaller than the wavelength used. In the solar spectrum (0.4 to 2  $\mu\text{m}$ ), reflection is therefore specular on smooth surfaces such as polished metal, still water or mirrors.

**NOTE:** While incoming solar light is unpolarized, reflected waves are generally polarized and Fresnel's laws can be used to determine polarization.

- Practically all real surfaces are not smooth and the surface reflection depends on the incident angle and the angle of reflection. Reflectance from such surfaces is referred to as **diffuse reflectance**.

**Bi-directional reflectance distribution function (BRDF)**,  $r(\mathbf{m}\mathbf{j}, -\mathbf{m}'\mathbf{j}')$  is introduced to characterize the angular dependence in the surface reflection and defined as the ratio of the reflected intensity to the energy flux in the incident beam:

$$r(\mathbf{m}\mathbf{j}, -\mathbf{m}'\mathbf{j}') = \frac{dI^\uparrow(\mathbf{t}_1, \mathbf{m}\mathbf{j})}{I^\downarrow(\mathbf{t}_1, -\mathbf{m}'\mathbf{j}') \mathbf{m}'\Omega'} \quad [19.1]$$

**NOTE:** Depending on a surface, **BRDF** has a specific spectral dependence. It plays a central role in the remote sensing of planetary surfaces.

**Reciprocity law:**  $r(\mathbf{m}\mathbf{j}, -\mathbf{m}'\mathbf{j}') = r(-\mathbf{m}'\mathbf{j}', \mathbf{m}\mathbf{j})$

- A surface called the **Lambert surface** if it obeys **the Lambert's Law**.

**Lambert's Law** of diffuse reflection: the diffusely reflected light is isotropic and unpolarized (i.e., natural light) independently of the state of polarization and the angle of the incidence light.

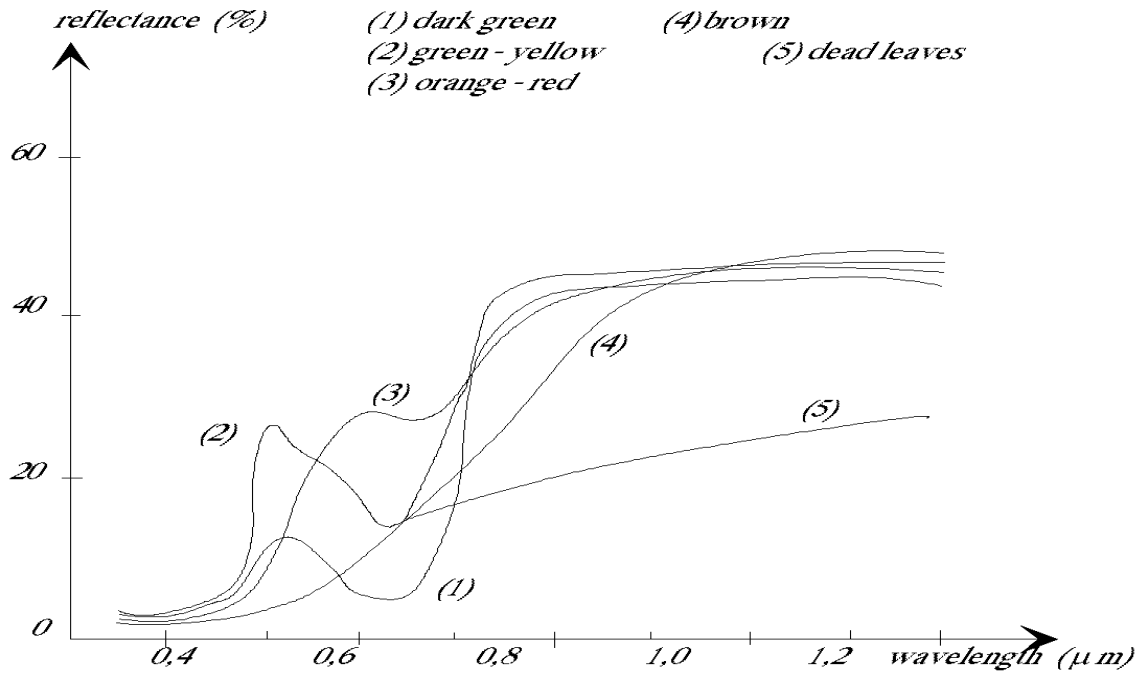
- **For the Lambert surface**, BRDF is independent on the directions of incident and observed light beam.

$$r(\mathbf{m}\mathbf{j}, -\mathbf{m}'\mathbf{j}') = r_{sur} \quad [19.2]$$

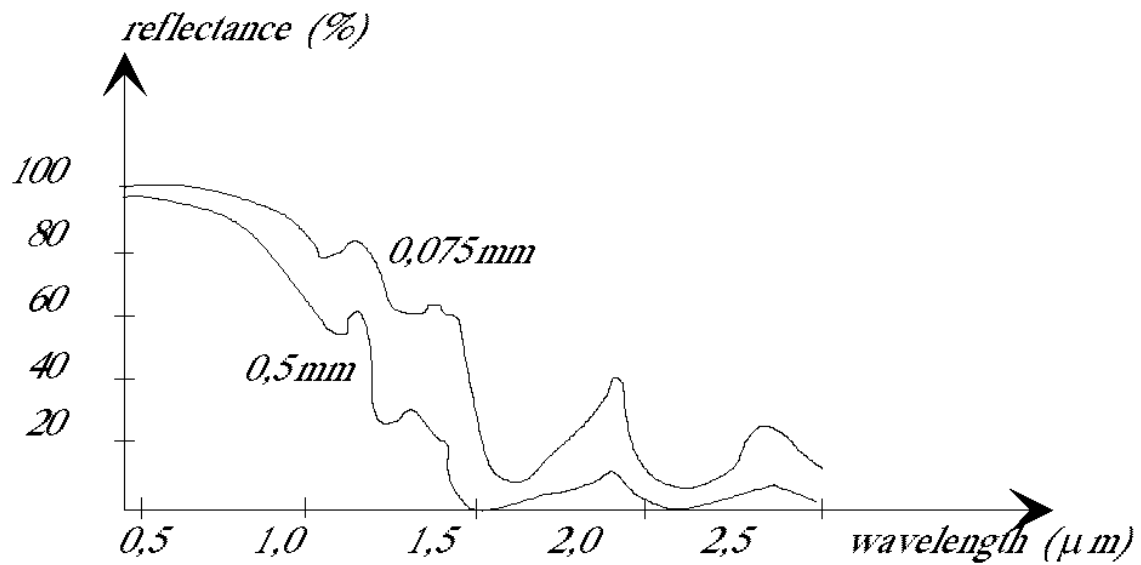
**NOTE:**  $r_{sur}$  is often called **surface albedo**. In general, it is a function of the wavelength. Examples of the surface albedo at about 550 nm: fresh snow/ice =0.8-0.9; desert=0.3, soils=0.1-0.25; ocean=0.05

**Examples of the spectral surface reflectance.**

**Figure 19.1** Evolution of the spectral reflectance of beech leaves during senescence (from Kniplinh, 1969).



**Figure 19.2** Reflectance of snow composed of crystals of  $d=0.075$  mm and  $d=0.5$  mm.



## 2. Inclusion of the surface reflection into the radiative transfer.

Let's include the contribution from the Lambert surface.

$$\text{Lambert surface: } I^\uparrow(\mathbf{t}_1, \mathbf{m}, \mathbf{j}) = I_{sur} = \text{const} \quad [19.3]$$

Generalizing the definitions for the reflection and transmission functions (i.e.,

Eqs.[18.1—[18.2]), we may express the reflected diffuse intensity  $I_r^\uparrow(0, \mathbf{m}, \mathbf{j})$  and

transmitted diffuse intensity  $I_t^\downarrow(\mathbf{t}_1, -\mathbf{m}, \mathbf{j})$  as

$$I_r^\uparrow(0, \mathbf{m}, \mathbf{j}) = \frac{1}{\mathbf{p}} \int_0^{2\mathbf{p}} \int_0^1 R(\mathbf{m}, \mathbf{j}, \mathbf{m}', \mathbf{j}') I_{inc}(-\mathbf{m}', \mathbf{j}') \mathbf{m}' d\mathbf{m}' d\mathbf{j}' \quad [19.4]$$

$$I_t^\downarrow(\mathbf{t}_1, -\mathbf{m}, \mathbf{j}) = \frac{1}{\mathbf{p}} \int_0^{2\mathbf{p}} \int_0^1 T(\mathbf{m}, \mathbf{j}, \mathbf{m}', \mathbf{j}') I_{inc}(-\mathbf{m}', \mathbf{j}') \mathbf{m}' d\mathbf{m}' d\mathbf{j}' \quad [19.5]$$

Let's consider the reflected diffuse intensity.

The reflected intensity at the top of the layer including the surface reflection may be written as

$$I^*(0, \mathbf{m}, \mathbf{j}) = I^\uparrow(0, \mathbf{m}, \mathbf{j}) + \frac{1}{\mathbf{p}} \int_0^{2\mathbf{p}} \int_0^1 T(\mathbf{m}, \mathbf{j}, \mathbf{m}', \mathbf{j}') I_{sur} \mathbf{m}' d\mathbf{m}' d\mathbf{j}' + I_{sur} \exp(-\mathbf{t}_1 / \mathbf{m}) \quad [19.6]$$

**NOTE:** Second term on the right-hand side gives the contribution from the surface reflected intensity which is diffusely transmitted to the top of the layer. Third term gives the contribution from the surface reflected intensity which is the directly transmitted.

Further, we can re-write Eq.[19.6] as

$$I^*(0, \mathbf{m}, \mathbf{j}) = \mathbf{m}_0 F_0 R(\mathbf{m}, \mathbf{j}, \mathbf{m}_0, \mathbf{j}_0) + I_{sur} \mathbf{g}(\mathbf{m}) \quad [19.7]$$

where

$$\mathbf{g}(\mathbf{m}) = \exp(-\mathbf{t}_1 / \mathbf{m}) + t(\mathbf{m})$$

Now, let's consider the diffuse transmitted intensity.

Isotropic intensity  $I_{sur}$ , propagating upward in the layer after being scattered by the Lambertian surface, can be partially reflected back to the surface and, hence, contribute to the downward intensity in the additional amount

$$I_{add}^{\downarrow}(-\mathbf{m}) = \frac{1}{p} \int_0^{2p} \int_0^1 R(\mathbf{m}, \mathbf{j}, \mathbf{m}', \mathbf{j}') I_{sur} m' dm' dj' = I_{sur} r(\mathbf{m})$$

Thus, the transmitted intensity including the surface contribution is

$$I^*(\mathbf{t}_1, -\mathbf{m}, \mathbf{j}) = I^{\downarrow}(\mathbf{t}_1, -\mathbf{m}, \mathbf{j}) + I_{sur} r(\mathbf{m}) = \mathbf{m}_0 F_0 T(\mathbf{m}, \mathbf{j}, \mathbf{m}_0, \mathbf{j}_0) + I_{sur} r(\mathbf{m}) \quad [19.8]$$

Both Eqs.[19.7] and [19.8] have  $I_{sur}$ . Thus, we need to find  $I_{sur}$ .

$$pI_{sur} = (\text{Surface albedo}) \times (\text{Downward flux})$$

**The downward flux has three components:**

(1) Transmitted direct flux =  $p\mathbf{m}_0 F_0 \exp(-\mathbf{t}_1 / \mathbf{m}_0)$

(2) Transmitted diffuse flux=

$$\int_0^{2p} \int_0^1 I^{\downarrow}(\mathbf{t}_1, -\mathbf{m}, \mathbf{j}) m dm dj = \int_0^{2p} \int_0^1 \mathbf{m}_0 F_0 T(\mathbf{m}, \mathbf{j}, \mathbf{m}_0, \mathbf{j}_0) m dm dj = p\mathbf{m}_0 F_0 t(\mathbf{m}_0)$$

(3) Fraction of  $I_{sur}$  reflected by the atmosphere back to the surface =

$$\int_0^{2p} \int_0^1 I_{add}^{\downarrow}(-\mathbf{m}) m dm dj = pI_{sur} \bar{r}$$

Therefore, we obtain that

$$pI_{sur} = r_{sur} (p\mathbf{m}_0 F_0 \exp(-\mathbf{t}_1 / \mathbf{m}_0) + p\mathbf{m}_0 F_0 t(\mathbf{m}_0) + pI_{sur} \bar{r})$$

and

$$I_{sur} = \frac{r_{sur}}{1 - r_{sur} \bar{r}} \mathbf{m}_0 F_0 \mathbf{g}(\mathbf{m}_0)$$

Therefore, the diffuse reflected and transmitted intensities, accounting for the surface contribution are

$$I^*(0, \mathbf{m}, \mathbf{j}) = I^\uparrow(0, \mathbf{m}, \mathbf{j}) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \mathbf{m}_0 F_0 \mathbf{g}(\mathbf{m}_0) \mathbf{g}(\mathbf{m}) \quad [19.9a]$$

$$I^*(t_1, -\mathbf{m}, \mathbf{j}) = I^\downarrow(t_1, -\mathbf{m}, \mathbf{j}) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \mathbf{m}_0 F_0 \mathbf{g}(\mathbf{m}_0) r(t_1, \mathbf{m}) \quad [19.9b]$$

Integrating Eq.[19.9a, b] over the solid angle, we find diffuse fluxes

$$F^*(0) = F^\uparrow(0) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \mathbf{m}_0 F_0 \mathbf{g}(\mathbf{m}_0) \bar{\mathbf{g}}$$

$$F^*(t_1) = F^\downarrow(t_1) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \mathbf{m}_0 F_0 \mathbf{g}(\mathbf{m}_0) \bar{r}$$

where

$$\bar{\mathbf{g}} = \bar{t} + 2 \int_0^1 \exp(-t_1 / \mathbf{m}_0) \mathbf{m}_0 d\mathbf{m}_0$$

NOTE:  $\bar{t}$  and  $\bar{r}$  were defined in Lecture 18 (see Eq.[18.8] and [18.9]).

NOTE: For non-Lambert surface, the inclusion of the surface reflection is a complex boundary problem.