

Lecture 22

Methods for solving the radiative transfer equation. Part 5: Monte Carlo method. Other methods.

Objectives:

1. Monte Carlo method.
2. Other methods for radiate transfer in inhomogeneous clouds.

Recommended reading:

G&Y: 8.4.3-8.4.4 and references below

Advanced reading:

Marchuk et al. The Monte Carlo methods in atmospheric optics. 1980

NOTE: Several Monte Carlo numerical codes are freely available at the WWW. For instance, MYSTIC (Monte Carlo code for the pYSically correct Tracing of photons In Cloudy atmospheres) is available at <http://www.libradtran.org/>

1. Monte Carlo method.

- The absorption and scattering processes in the atmosphere can be considered as stochastic processes.

Phase function can be interpreted as a probability function for the redistribution of photons in different directions.

Single scattering albedo ω_0 can be interpreted as the probability that a photon will be scattered, given an extinction event.

NOTE: $1 - \omega_0$ is called **co-albedo** and can be considered as the probability of absorption per extinction event.

Recall that energy of one photon is hc/λ , where $h = 6.626 \times 10^{-34}$ J s

Solar flux at the top of the atmosphere at 550 nm = 2.55×10^{15} photons $\text{cm}^{-2} \text{s}^{-1}$

Thus, the radiative field can be predicted by statistical analysis of traveling photons.

Generation of random numbers:

Using a random number generator (numerical algorithm), the random numbers, rn , between 0 and 1 with a probability distribution function $PDF = 1$ can be generated.

Using this rn , we can generate another set of random numbers as

$$rx = -\ln(rn)$$

with $PDF = \exp(-rx)$ and rx between 0 and infinity.

Let's consider a homogeneous medium characterized by the extinction coefficient β_{ext} , single scattering albedo ω_0 and phase function $P(\mu, \mu')$.

Monte Carlo method simulates the trajectories of individual photons according to the following scheme:

- (1) Determine starting position x_0 and direction (μ, ϕ) of a photon
- (2) Generate a photon path length (using random numbers rx)

$$x = rx$$

- (3) Calculate a new photon position (x_0+x) called the event point (or collision point)
- (4) Analyze what can happen with the photon at this event point by generating the random number rn and comparing it with ω_0

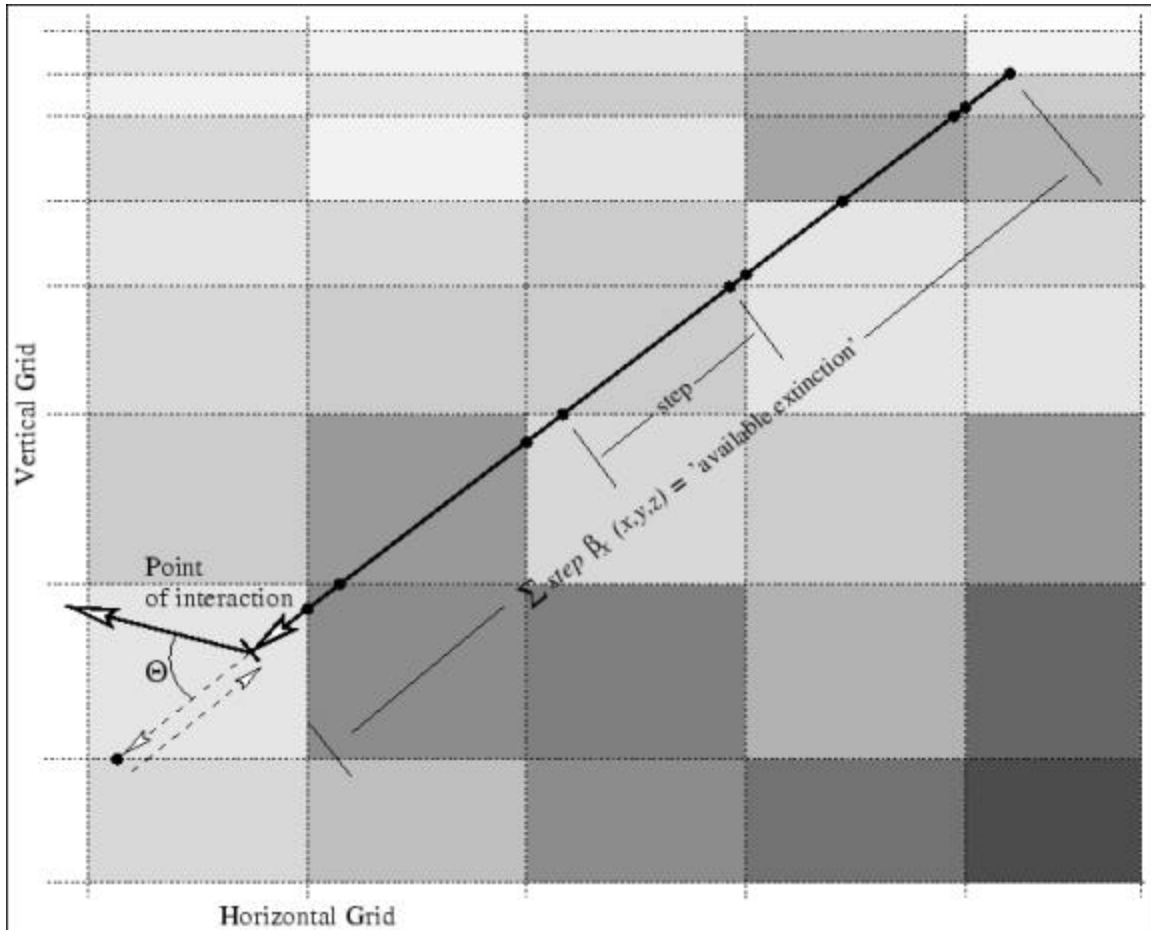
if $rn > \omega_0 \Rightarrow$ the photon is absorbed \Rightarrow go to (1) for a new photon.

if $rn < \omega_0 \Rightarrow$ the photon is scattered \Rightarrow go to (5)

- (5) Find a new direction for the scattered photon using the phase function to calculate the cumulative probability function to relate the scattering angle to a random number.
- (6) Then repeat starting with (3) until the all photons are analyzed.

- Monte Carlo requires about $10^6 - 10^6$ photons to produce statistically reliable results.
- **Backward Monte Carlo method:** starts with the photon at the detector and traces back to the source.

Let's consider the inhomogeneous atmosphere. We can split it into the homogeneous grids.



Point of interaction (or event point)

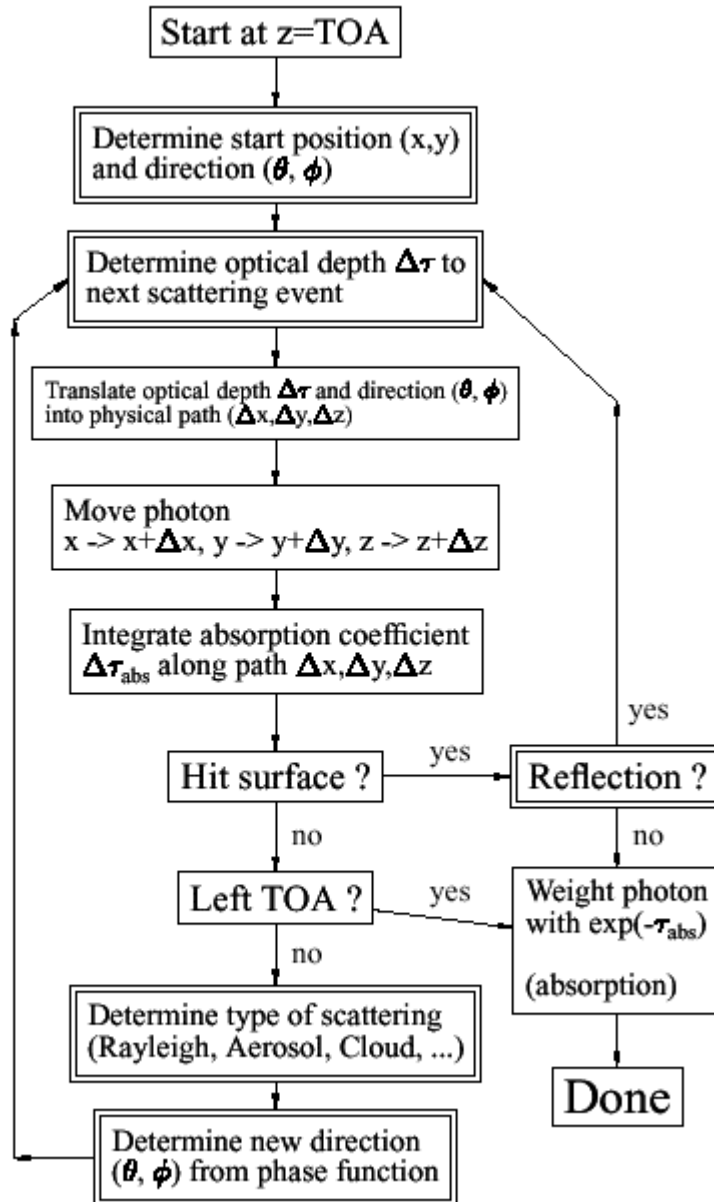
$$rx = \sum_{\text{step}} \text{step } \mathbf{b}_{\text{ext}}(x, y, z)$$

where step = step-size in each grid

➤ $l = 1/b_{\text{ext}}$ is called **free path-length**.

$$\frac{I}{I_0} = \exp(-\tau) = \exp(-\beta_{\text{ext}} x) = \exp(-x/l) \quad [22.1]$$

Realization of the Monte Carlo method in the MYSTIC model.



NOTE: MYSTIC model uses a 'weight' technique to account for absorption. It helps to speed up the calculations.

2. Other methods.

How to treat the inhomogeneity of clouds:

Independent Pixel Approximation (IPA)

(Cahalan et al., The albedo of fractal stratocumulus clouds. *J. Atmos. Sci.*, 51, 2434-2455, 1994)

- IPA is computational efficient technique to calculate the radiative transfer accounting for the cloud inhomogeneity. A cloud is subdivided into columns, plane-parallel radiative transfer is applied to each column, and the overall radiative transfer effect is the summation from the individual columns. Thus, IPA calculates the domain-averaged radiative properties.
- IPA concept is well suitable for GCM models, but does not work well in the remote sensing of cloud properties.
- Modification of IPA, NIPA (nonlocal independent pixel approximation) has been proposed to account for ‘radiative smoothing’ effect (i.e., the tendency of horizontal photon transport to smooth the radiative field predicted by IPA).

Spherical Harmonic Discrete Ordinate Method (SHDOM):

(developed by F. Evans, <http://nit.colorado.edu/~evans/shdom.html>)

- SHDOM is a highly efficient and flexible 3D atmospheric radiative transfer model.
- SHDOM uses an iterative process to compute the source function (including the scattering integral) on a grid of points in space. The angular part of the source function is represented with a spherical harmonic expansion.
- SHDOM can compute unpolarized, monochromatic and broadband (with a k -distribution), shortwave and longwave radiative transfer. The medium properties (extinction, phase function, etc.) are specified at each grid point, and the surface albedo may vary as well.

Figure 22.1 Comparison of Monte Carlo and SHDOM methods: differences (SHDOM-MC) in radiative fluxes (at 1.65 μm) reflected by the inhomogeneous cloud (Evans et al.)

