Lecture 23

Review for mid-term exam 2: Solar radiative transfer processes

Topics to review:

- **Concepts of scattering, polarization, scattering phase function and Stoke parameters** (Lecture 13). Qualitative understanding of Eqs.[13.6]; [13.8]; [13.11]

Recall that

$$I \sim |E|^2 \qquad [13.2]$$

The electric vector $\vec{E}$ may be decomposed into the parallel E_l and perpendicular E_r components as

$$E_l = a_l \exp(-id_l)\exp(-ikz + ivt) \qquad [13.2a]$$

$$E_r = a_r \exp(-id_r)\exp(-ikz + ivt) \qquad [13.2b]$$

where a_l and a_r are the positive **amplitude** of the parallel E_l and perpendicular E_r components, respectively; d_l and d_r are the positive **phases** of the parallel E_l and perpendicular E_r components, respectively; ***k*** is the propagation (or wave) constant, ***k* = 2**p/l w is the circular frequency, w = ***kc=2*p *c*/*l***

Stokes parameters: so-called intensity I, the degree of polarization Q, the plane of polarization U, and the ellipticity V of the electromagnetic wave.

$$I = E_l E_l^* + E_r E_r^*$$
$$Q = E_l E_l^* - E_r E_r^* \qquad [13.6]$$
$$U = E_l E_r^* + E_r E_l^*$$
$$V = -i(E_l E_r^* - E_r E_l^*)$$

$$I = a_l^2 + a_r^2$$
$$Q = a_l^2 - a_r^2 \qquad [13.8]$$
$$U = 2a_l a_r \cos(d)$$
$$V = 2a_l a_r \sin(d)$$

The degree of linear polarization *LP* of a light beam is defined (neglecting U and V) as

$$LP = -\frac{Q}{I} = -\frac{I_l - I_r}{I_l + I_r} \qquad [13.10]$$

➢ **Rayleigh (molecular) scattering** (Lecture 13: Eqs.[13.12]; [13.13]; [13.15]; [13.16])

Rayleigh scattering: x=2πr/λ <<1, m is arbitrary (applies to scattering by molecules and small particles)

Mie-Debye scattering: x=2πr/λ and m are both arbitrary but for spheres only (applies to scattering by aerosol and cloud particles)

Rayleigh scattering:

$$I_r = I_{0r} k^4 \alpha^2 / r^2$$

$$I_l = I_{0l} k^4 \alpha^2 \cos^2(\Theta) / r^2$$

thus

$$I = I_r + I_l = \frac{I_0}{r^2} \alpha^2 \left(\frac{2\pi}{\lambda}\right)^4 \frac{1+\cos^2(\Theta)}{2} \qquad [13.12]$$

Rayleigh scattering **phase function** for incident unpolarized radiation

$$P(\cos(\Theta)) = \frac{3}{4}(1+\cos^2(\Theta)) \qquad [13.13]$$

and **asymmetry factor g=0** or Rayleigh scattering.

NOTE: ***General definition***: the phase function P(cosΘ) is defined as a non-dimensional parameter to describe the angular distribution of the scattered radiation as

$$\frac{1}{4\pi} \int_{\Omega} P(\cos\Theta) d\Omega = 1 \qquad [13.11]$$

where Θ is called the **scattering angle** between the direction of incidence and observation ($\cos(\Theta) = \cos(\theta')\cos(\theta) + \sin(\theta')\sin(\theta)\cos(\phi'-\phi)$)

Eq.[13.12] may be re-written as

$$I(\cos(\Theta)) = \frac{I_0}{r^2} \sigma_s \frac{P(\Theta)}{4\pi} \qquad [13.15]$$

where $\sigma_s = \alpha^2 \frac{128\pi^5}{3\lambda^4}$

Optical depth of Rayleigh scattering of an atmospheric layer between Z1 and Z2:

$$t(l) = s_s(l) \int_{z_1}^{z_2} N(z)dz$$

- **Single-scattering and absorption by aerosol and cloud particles** (Lectures 14-15):

- Outline of Mie theory. Concepts of extinction, absorption and scattering efficiencies of an individual particle.
- Optical properties of the ensemble of particles (i.e., particles of various sizes and compositions)
- Parameterizations of the optical properties of cloud droplets (Eqs.[15.1]-[15.4])
- Basic principles of measurements of light scattering and absorption by particulates. Measurements of optical depth.

Assumptions in Mie theory

i) Particle is a **sphere** of radius ***a***;

ii) Particle is characterized by a **single refractive index** $\boldsymbol{m=m_r - im_i}$ at a given wavelength;

NOTE*:* see the strategy to compute optics using Mie theory in Lecture15

Mie theory gives the extinction, scattering and absorption cross-sections (and efficiencies) and the scattering phase matrix of a single spherical particle with a size parameter $\boldsymbol{x = 2p r/l}$

If the particles characterized by a size distribution N(r), the extinction, scattering and absorption coefficients (in units LENGTH^{-1}) are calculated as

$$b_e = \int_{r_1}^{r_2} s_e(r)N(r)dr\,;\quad b_s = \int_{r_1}^{r_2} s_s(r)N(r)dr\,;\quad b_a = \int_{r_1}^{r_2} s_a(r)N(r)dr$$

The **single scattering albedo** if defined as $v_0 = \frac{b_s}{b_e}$ [14.12]

Water drops

The **effective radius** is defined as

$$r_e = \frac{\int pr^3 N(r)dr}{\int pr^2 N(r)dr} \qquad [15.1]$$

The extinction coefficient (for large drops when $Q_e = 2$)

$$b_e \approx \frac{3}{2}\frac{LWC}{r_e} \qquad [15.3]$$

- **Concepts of diffuse and direct solar radiation in the atmosphere** (Lecture 17)

- Attenuation of the direct solar radiation (Eqs. .[17.1]-[17.2])
- Diffuse radiation. Concept of multiple scattering. Source function for diffuse solar radiation. (Eq.[17.3]). General form of the radiative transfer equation for the diffuse radiation (Eq.[17.4]).

Direct solar radiation is a part of solar radiation filed that has survived the extinction passing a layer with optical depth t***** and it obeys the Beer-Bouguer-Lambert (extinction) law:

$$F_{dir}^{\downarrow} = m_0 p F_0 \exp(-t^* / m_0) \qquad [17.2]$$

where $F_{dir}^{\downarrow}$, $\boldsymbol{F_0}$ and t***** are the functions of wavelength.

The **source function for diffuse solar radiation** may be written as two components

$$J(t, m, j) = \frac{v_0}{4p}\int_0^{2p}\int_{-1}^{1} I(t, m', j')P(m, j, m', j')dm'dj' + \\ + \frac{v_0}{4p} pF_0 P(m, j, -m_0, j_0)\exp(-t / m_0) \qquad [17.3]$$

where the w_0 is the single scattering albedo and $\boldsymbol{P}$ is the scattering phase function.

Using the source function for scattering, we can write the **radiative transfer equation for the <u>diffuse</u> radiation** as

$$m\frac{dI(t,\vec{\Omega})}{dt} = I(t,\vec{\Omega}) - \frac{v_0}{4p}\int_{4p} I(t,\vec{\Omega}')P(\vec{\Omega},\vec{\Omega}')d\Omega' - \frac{w_0}{4p}pF_0 P(\vec{\Omega},-\vec{\Omega}_0)\exp(-t/m_0)$$

[17.4]

- **Methods for solving radiative transfer equation with multiple scattering and absorption:**

Approximate methods:

i) Single scattering approximations (Lecture 17)

ii) Two-stream approximations (Lecture 18 and Homework 3)

iii) Eddington and Delta- Eddington approximations (Lecture 18 and Homework 3)

"Exact" methods:

i) Discrete-ordinate technique (Lecture 20)

ii) Adding-doubling technique (Lecture 21)

iii) Monte-Carlo technique (Lecture 22)

<u>Single scattering approximations</u>: the **source function** is

$$J(t,m,j) = \frac{v_0}{4p}pF_0 P(m,j,-m_0,j_0)\exp(-t/m_0)$$

the **solutions for reflected and transmitted diffuse intensities**

$$I_l^{\uparrow}(0;m;j) = \frac{v_0 m_0 F_0}{4(m+m_0)}P(m,j,-m_0,j_0)\left[1-\exp\left(-t_1\left(\frac{1}{m}+\frac{1}{m_0}\right)\right)\right]$$

[17.9]

and for m is NOT equaled to m_0

$$I_l^{\downarrow}(t_1;-m;j) = \frac{v_0 m_0 F_0}{4(m-m_0)}P(-m,j,-m_0,j_0)\left[\exp\left(-\frac{t_1}{m}\right)-\exp\left(-\frac{t_1}{m_0}\right)\right]$$

[17.10a]

and for $m=m_0$

$$I_{\lambda}^{\downarrow}(\tau_1;-\mu;\varphi) = \frac{\omega_0 \tau_1 F_0}{4\mu_0} P(-\mu_0, \varphi_0, -\mu_0, \varphi_0)\left[\exp\left(-\frac{\tau_1}{\mu_0}\right)\right] \qquad [17.10b]$$

NOTE: For the single scattering approximation, the diffuse intensities are directly proportional to the phase function.

- **Concepts of the reflection and transmission of an atmospheric layer** (Lecture 18: Eqs.[18.1]-[18.9])

Concepts of the reflection function, planetary reflectivity and global albedo

Concepts of the diffuse transmission function, transmission and global transmission.

- **Reflection from the surface** (Lecture 19: Eqs.[19.1]-[19.2])

Concept of the Lambert surface