Lecture 6

Terrestrial infrared radiative processes. Part 2:

Gaseous absorption/emission: Line-by-line approach.

K-distribution approximation.

Objectives:

1. Line-by-line computations of radiative transfer in IR.
2. K-distribution approximation (KD).
3. Correlated k-distribution approximation (CKD).

Required reading:

L92: 2.3.3; 2.6

Recommended reading:

G&Y: 6.1; 6.2.4

1. Line-by-line (LBL) computations of radiative transfer in IR.

LBL method is considered to be an "exact" computation of radiative transfer in the gaseous absorbing/emitting inhomogeneous atmosphere, accounting for all (known) gas absorption lines in the wavenumber range from 0 to about 23,000 cm^{-1}.

Recall an **equation of radiative transfer** (Eq.[3.9a, b], Lecture 3) in **the plane-parallel atmosphere**

$$m\frac{dI_l^{\uparrow}(t;m;j)}{dt} = I_l^{\uparrow}(t;m;j) - J_l^{\uparrow}(t;m;j) \qquad [3.9a]$$

$$-m\frac{dI_l^{\downarrow}(t;-m;j)}{dt} = I_l^{\downarrow}(t;-m;j) - J_l^{\downarrow}(t;-m;j) \qquad [3.9b]$$

and its **solutions** (Eq.[3.10a,b]) derived in Lecture 3:

$$I_l^{\uparrow}(t;m;j) = I_l^{\uparrow}(t_1;m;j)\exp(-\frac{t_1 - t}{m})$$

$$+\frac{1}{m}\int_{t}^{t_1}\exp(-\frac{t' - t}{m})\ J_l^{\uparrow}(t';m;j)dt' \qquad [3.10a]$$

$$I_l^{\downarrow}(t;-m;j) = I_l^{\downarrow}(0;-m;j)\exp(-\frac{t}{m})$$

$$+\frac{1}{m}\int_{0}^{t}\exp(-\frac{t - t'}{m})\ J_l^{\downarrow}(t';-m;j)dt' \qquad [3.10b]$$

For infrared radiative transfer:

non-scattering medium in the local thermodynamical equilibrium:

the **source function** is given by Plank's function $B_l(T)$,

and $b_{e,l} = b_{a,l} = k_l r$, where k_l is the mass absorption coefficient.

Assuming that the thermal infrared radiation from the earth's atmosphere is independent on the azimuthal angle ϕ, **the equation of infrared radiative transfer for the monochromatic upward and downward intensities** can be expressed in the wavenumber domain as

$$m\frac{dI_n^{\uparrow}(t;m)}{dt} = I_n^{\uparrow}(t;m) - B_n(T) \qquad [6.1a]$$

$$-m\frac{dI_n^{\downarrow}(t;-m)}{dt} = I_n^{\downarrow}(t;-m) - B_n(T) \qquad [6.1b]$$

and **its solutions** as

$$I_n^{\uparrow}(t\,;m) = I_n^{\uparrow}(t_1;m)\exp(-\frac{t_1 - t}{m})$$

$$+\frac{1}{m}\int_{t}^{t_1}\exp(-\frac{t' - t}{m})\; B_n(T(t'))\,dt' \qquad [6.2a]$$

$$I_n^{\downarrow}(t\,;-m) = I_n^{\downarrow}(0;-m)\exp(-\frac{t}{m})$$

$$+\frac{1}{m}\int_{0}^{t}\exp(-\frac{t - t'}{m})\; B_n(T(t'))\,dt' \qquad [6.2b]$$

If Eqs.[6.2a, b] are to be solved for the whole atmosphere with total optical depth t_n^*, two boundary conditions are required.

Surface: assumed to be a blackbody in the IR emitting with the surface temperature T_s,

$$I_n^{\uparrow}(t_n^*, m) = B_n(T_s) = B_n(T_s(t_n^*)) = B_n(t_n^*)$$

Top of the atmosphere (TOA), $t_n = 0$: no downward emission

$$I_n^{\downarrow}(0, -m) = B_n(TOA) = 0$$

Using the above boundary conditions, **the formal solutions for monochromatic upward and downward intensities are**

$$I_\nu^{\uparrow}(\tau;\mu) = B_\nu(\tau^*)\exp\left(-\frac{\tau^* - \tau}{\mu}\right) + \frac{1}{\mu}\int_{\tau}^{\tau^*} \exp\left(-\frac{\tau' - \tau}{\mu}\right) B_\nu(\tau')\,d\tau' \qquad [6.3a]$$

$$I_\nu^{\downarrow}(\tau;-\mu) = \frac{1}{\mu}\int_{0}^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) B_\nu(\tau')\,d\tau' \qquad [6.3b]$$

To solve Eqs. [6.3a, b] “exactly”, one needs to compute a monochromatic optical depth τ_ν (due to absorption of atmospheric gases).

This is done by using the LBL method.

NOTE: Labs 3-4 are devoted to LBL computations.

The formal solutions for monochromatic upward and downward intensities can be also expressed in terms of **monochromatic transmittance** (defined in Lecture 5).

By definition, the monochromatic transmittance is

$$T_n(t/m) = \exp(-\frac{t_n}{m})$$

and the differential form is

$$\frac{dT_n(t/m)}{dt} = -\frac{1}{m}\exp(-\frac{t_n}{m})$$

Thus **the formal solutions for monochromatic upward and downward intensities** given by Eq.[6.3a,b] in terms of transmittance are:

$$I_n^{\uparrow}(t;m) = B_n(t^*)T_n((t^* - t)/m) - \int_t^{t^*} B_n(t')\frac{dT_n((t' - t)/m)}{dt'}dt' \qquad [6.4a]$$

$$I_n^{\downarrow}(t;-m) = \int_0^{t} B_n(t')\frac{dT_n((t - t')/m)}{dt'}dt' \qquad [6.4b]$$

Strategy to perform LBL calculations to solve Eq.[6.3a,b] for the plane-parallel atmosphere:

For a given wavenumber n:

For the j-th atmospheric layer:(homogeneous; temperature T_j, pressure $p_{j,}$ length ΔZj)

For n-th gas:

Absorption coefficient $k_{n,j,n}$ is

$$k_{n,j,n} = \sum_{l=1}^{L} k_{n,j,n,l} = \sum_{l=1}^{L} S_{n,n,l}(T_j) f_{n,n,l}(T_j, p_j)$$

where l = 1, ..L in the number of absorbing lines of n-th gas at a selected ν; $S_{n,n,l}$ and $f_{n,n,l}$ are the intensity and profile of the l-th line.

Optical depth $t_{n,j,n,}$ of n-th gas of j-th layer

$$t_{n,j,n} = k_{n,n,j} u_{n,j}$$

where $u_{n,j}$ is the slant path for n-gas in j-th layer (i.e., the amount of n-th gas in j-th layer).

Repeating above calculations for all gases n=1, ...N, we find optical depth of j-th layer

$$t_{n,j} = \sum_{n=1}^{N} t_{n,j,n}$$

Repeating above calculations for all layers j=1, ...J, we find optical depth of each layer.

Using calculated optical depth of each layer we find the **monochromatic upward and downward intensities** from Eq.[6.3a,b].

NOTE: Similar strategy is used to solve Eq.[6.4a,b] via monochromatic transmission function:

for n-th gas

$$T_{n,j,n} = \exp(-\frac{t_{n,j,n}}{m})$$

and, using the **multiplication law** of transmittance, we have the transmittance of j-th layer of N absorbing gases

$$T_{n,j} = T_{n,1} T_{n,2} \ldots T_{n,N}$$

- **Multiplication law of transmittance** states that when several gases absorb, the monochromatic transmittance is a product of the monochromatic transmittances of individual gases:

$$T_{n,1,2\ldots N} = T_{n,1} T_{n,2} \ldots T_{n,N}$$

❖ **For all practical applications, one needs to know not monochromatic intensity (or flux) but intensity (or flux) averaged over a given wavenumber interval.**

Spectral intensity = intensity averaged over a very narrow interval that B_ν is almost constant but the interval is large enough to consist of several absorption lines.

Narrow-band intensity= intensity averaged over a narrow band which includes a lot of lines;

Broad-band intensity= intensity averaged over a broad band (e.g., over a whole longwave region)

We can define the spectral **transmission function** for a band of a width **Dn** as

$$T_{\bar{n}}(u) = \frac{1}{\Delta n} \int_{\Delta n} T_n(t) dn = \frac{1}{\Delta n} \int_{\Delta n} \exp(-t_n) dn = \frac{1}{\Delta n} \int_{\Delta n} \exp(-k_n u) dn$$

[6.5]

NOTE: **Spectral intensify** requires the calculations of **spectral transmission** which requires the calculations of **monochromatic optical depth** which are done with LBL computations.

<u>LBL spectral resolution:</u>

- Because LBL computes each line of absorbing gases in non-homogeneous atmopshere, the adequate selection of an integration step (i.e., interval ***d*n)** is required to calculate the spectral transmittance in the interval **Dn** (**Dn >** ***d*n).**
- Because P decreases exponentially with altitude, the line half-width and hence the integration step should be smaller at higher altitudes in the atmosphere.
- Because of these variable resolutions, the absorptions coefficients of two consequent layers must be merged – it is done by interpolating the coarser-resolution of lower layers into the finer-resolution of the higher-level => spectral absorptance for a given slant path is computed with the finest spectral resolution.

NOTE: Absorption lines may have long wings (e.g., depending on a line half-width). To simplify calculations, the wings of a line are cut at a given distance from the line center Thus the absorption coefficient of the line may be expressed as

$$k_n = \sum_{l=1}^{L} S_n(T)\, f_n(T, p) + k_n^c \qquad [6.6]$$

where k_n^c gives the absorption fraction in the wings (called **continuum absorption**). (See Lab 4 for further details).

2. K-distribution method (KD).

KD method is developed to compute the **spectral transmittance** (hence the spectral intensity or spectral fluxes) based on grouping of gaseous absorption coefficients.

- KD method is benefit from the fact that the same value of k_ν is encountered many times over a given spectral interval => thus to eliminate the redundancy, one can group the values of k and perform the transmittance calculation only once for a given value of k.

Consider a **homogeneous** atmospheric layer. In this case, the spectral transmittance is independent of the ordering of k in the spectral interval **Dn**.

We can introduce the normalized **probability distribution function** ***f(k)***

$$f(k) = \frac{1}{\Delta n}\frac{dn}{dk} = \frac{1}{\Delta n}\sum_j \left| \frac{\Delta n_j}{\Delta k} \right|$$

where **Dn**$_{j.}$ is the subinterval of **Dn** where ***k*** is a monotonic function of **n,** and $\int_0^\infty f(k) = 1$

Then the **cumulative probability function** can be defined as

$$g(k) = \int_0^k f(k) dk$$

and g(0)=0; g(∞)=1 and ***dg(k)=f(k)dk.***

NOTE: By definition, g(k) is a monotonically increasing and smooth function in k-space, Therefore, k(g), as an inverse function of g(k), is a smooth function in g-space.

Therefore the **spectral transmittance** can be written as

$$T_{\bar{n}}(u) = \frac{1}{\Delta n}\int_{\Delta n} \exp(-k_n u) dn = \int_0^\infty \exp(-ku) f(k) dk = \int_0^1 \exp(-k(g)u) dg$$

Because both g(k) and k(g) are smooth functions, the above integral can be calculated by a finite sum as

$$T_{\bar{n}}(u) = \int_0^1 \exp(-k(g)u)dg \approx \sum_{i=1}^{N} \exp(-k(g_i)u)\Delta g_i =$$

$$= \Delta g_1 e^{-k_1 u} + \Delta g_2 e^{-k_2 u} + \ldots + \Delta g_N e^{-k_N u}$$

where Δg_i is the quadrature weight.

Thus the **KD method** allows calculating the spectral transmittance as a finite weighted sum of exponent in g-space, replacing the tedious wavenumber integration.

Numerical realization of KD:

(see illustration below)

Consider a spectral interval $\Delta\nu$ that contains numerous absorption lines.

Let's divide it into N intervals of $\Delta\nu_j$, j = 1,2,3…N

The probability distribution function can be written as

$$f(k) = \frac{1}{\Delta n}\frac{dn}{dk} = \frac{1}{\Delta n}\sum_j \left|\frac{\Delta n_j}{\Delta k}\right|$$

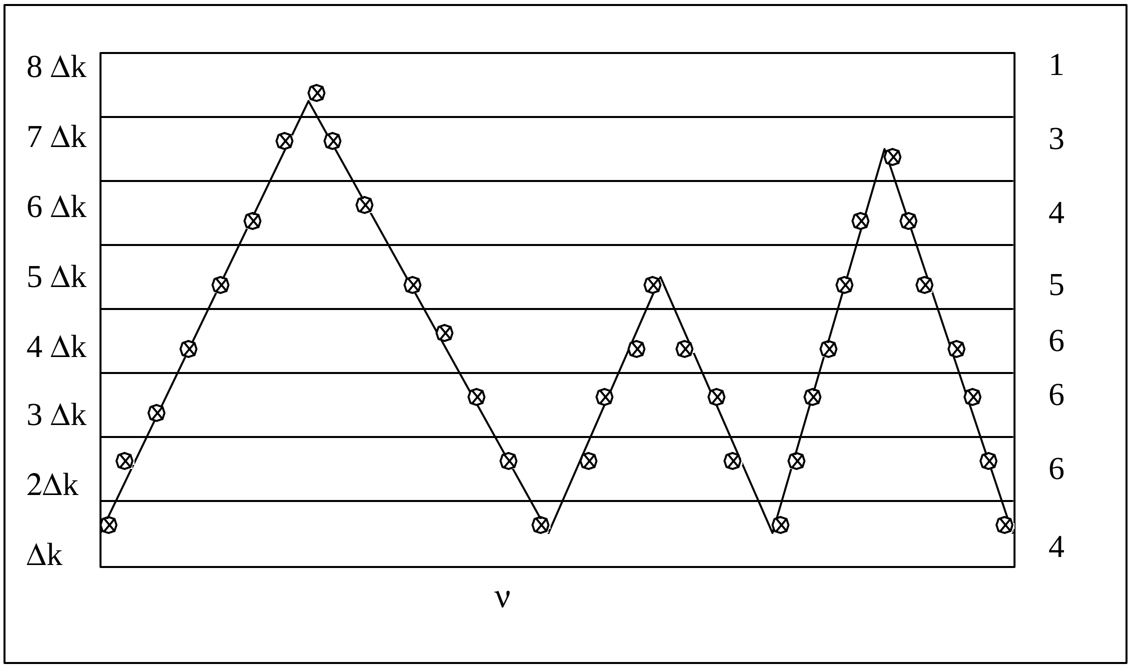
where $\Delta\nu_j$. is the subinterval of $\Delta\nu$ where k is a monotonic function of ν.

Then the cumulative probability is

$$g(k) = \frac{1}{\Delta n}\sum_j \int_0^k \left|\frac{\Delta n_i}{\Delta k'}\right| dk' = \frac{1}{\Delta n}\sum_j \int_0^k \Delta n_j(k) = \frac{n(0,k)}{N}$$

where ***n(0,k)*** is the number of computational points that contribute to ***k*** cumulatively.

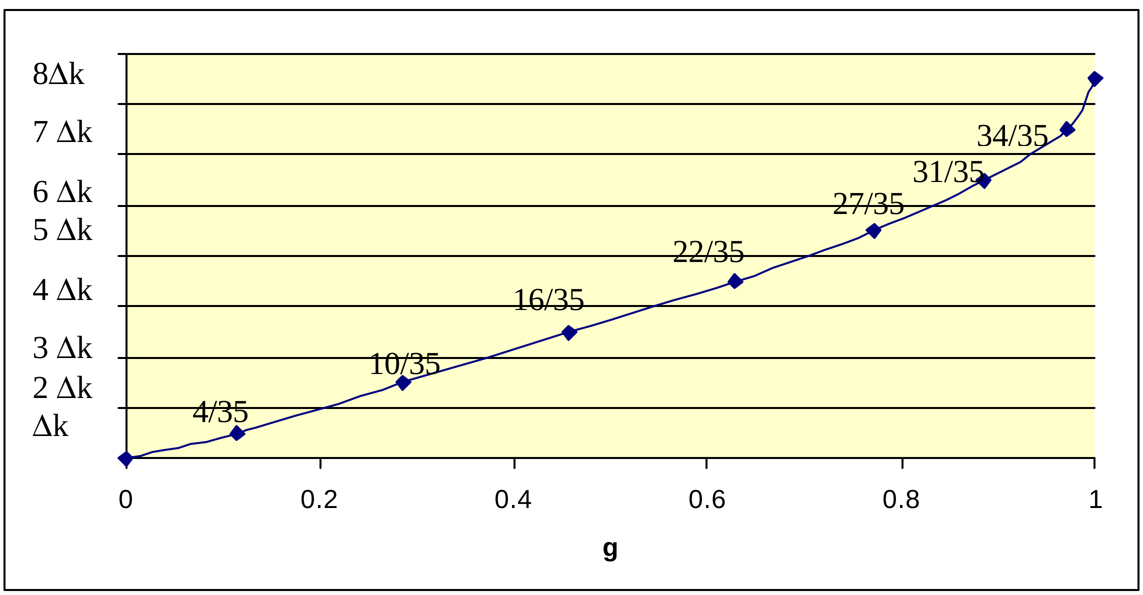
Figure 6.1 How to calculate the absorption coefficient in g-space from the known absorption coefficient in the wavenumber domain.



Solid line gives absorption coefficient as a function of ν.

Numbers on the right side are the data points in each Δk interval (total number N=35).

Thus by definition, $g(j\Delta k)=n(0, j\Delta k)/N$



3. Correlated K-distribution approximation (CKD).

CKD is the extension of KD for inhomogeneous atmosphere.

NOTE: CKD is discussed in details in several research papers. As a part of your Homework 2, you need to study the paper by

Fu, Q., and K.N. Liou, On the correlated k-distribution method for radiative transfer in nonhomogeneous atmospheres. *Journal of the Atmospheric Sciences* 49, 2139-2156, 1992.