Lecture 8

Terrestrial infrared radiative processes. Part 4:

Infrared radiative transfer in the plane-parallel atmosphere.

IR radiative heating/cooling rates

Objectives:

1. Infrared radiative transfer in the plane-parallel atmosphere.
2. Infrared radiative heating/cooling rates.
3. Concept of broadband flux emissivity.

Required reading:

L80: 1.4.4; 4.3; 4.7; 4.8.1

Recommended/advanced reading:

Le93:11 G&Y: 6.3-6.4

1. Infrared radiative transfer in the plane-parallel atmosphere.

- Recall Lecture 6 where we have derived the solutions of a radiative transfer for the **monochromatic upward and downward intensities** in the IR for a plane-parallel atmosphere consisting of absorbing gases (no scattering => no clouds, no aerosols) (see Eq.[6.3a, b] and Eq.[6.4a, b]).

$$I_n^{\uparrow}(t;m) = B_n(t^*)\exp\left(-\frac{t^*-t}{m}\right) + \frac{1}{m}\int_t^{t^*}\exp\left(-\frac{t'-t}{m}\right) B_n(t')dt' \quad [6.3a]$$

$$I_n^{\downarrow}(t;-m) = \frac{1}{m}\int_0^{t}\exp\left(-\frac{t-t'}{m}\right) B_n(t')dt' \quad [6.3b]$$

and

$$I_n^{\uparrow}(t\,;m) = B_n(t^{*})T_n((t^{*}-t)/m)$$

$$-\int_{t}^{t^{*}} B_n(t')\frac{dT_n((t'-t)/m)}{dt'}dt' \qquad [6.4a]$$

$$I_n^{\downarrow}(t\,;-m) = \int_{0}^{t} B_n(t')\frac{dT_n((t-t')/m)}{dt'}dt' \qquad [6.4b]$$

Recall that Eq.[6.3a, b] and Eq.[6.4a, b] have been derived the whole atmosphere with the optical depth τ_ν^* assuming two boundary conditions:

Surface: assumed to be a blackbody in the IR emitting with the surface temperature T_s,

$$I_n^{\uparrow}(t_n^{*},m) = B_n(T_s) = B_n(T_s(t_n^{*})) = B_n(t_n^{*})$$

Top of the atmosphere (TOA), t_n **= 0:** no downward emission

$$I_n^{\downarrow}(0,m) = B_n(TOA) = 0$$

- **For various practical applications we need to know the radiative flux (or irradiance).**

We can introduce the **monochromatic upward and downward fluxes** by using the general definition for a monochromatic radiative flux (see Lecture 2, Eq.[2.6])

$$F_l = \int_{\Omega} I_l \cos(q)\, d\Omega \qquad [2.6]$$

and that $d\Omega = \sin(q)\, dq\, dj$

Therefore we have

$$F_n^{\uparrow} = \int_0^{2p}\int_0^{p/2} I_n^{\uparrow}(q;j)\cos(q)\sin(q)\,dq\,dj =$$

$$= -2p \int_0^{p/2} I_n^{\uparrow}(q)\cos(q)\,d(\cos(q)) =$$

$$= 2p \int_0^{1} I_n^{\uparrow}(m)\,m\,dm$$

NOTE: above we assumed that there is no dependency of f in the plane-parallel atmosphere. Also we used that ***d(cos**q**) = - sin(**q**)d**q*

Performing the similar derivation for $F_n^{\downarrow}$, we have

$$F_n^{\uparrow} = 2p \int_0^{1} I_n^{\uparrow}(m)\,m\,dm$$

$$F_n^{\downarrow} = 2p \int_0^{1} I_n^{\downarrow}(-m)\,m\,dm \qquad [8.1]$$

Thus we can re-write the radiative transfer equation and its solution (Eq.[6.1a, b] and Eq.[6.3a, b]) in terms of **the monochromatic upward and downward fluxes.**

Similar to Eq.[6.3a, b] we have

$$F_n^{\uparrow}(t) = 2p B_n(t^*) \int_0^1 \exp\left(-\frac{t^* - t}{m}\right) m\, dm + 2p \int_0^1 \int_t^{t^*} \exp\left(-\frac{t' - t}{m}\right) B_n(t') dt' dm \qquad [8.2a]$$

and

$$F_n^{\downarrow}(t) = 2p \int_0^1 \int_0^t \exp\left(-\frac{t - t'}{m}\right) B_n(t') dt' dm \qquad [8.2b]$$

- To integrate Eq.[8.2a, b] over m, let's introduce the **exponential integral** defined as

$$E_n(t) = \int_1^{\infty} \frac{\exp(-tx)}{x^n} dx$$

NOTE:

$$\frac{dE_n(t)}{dt} = -\int_1^{\infty} \frac{\exp(-tx)}{x^n} = -E_{n-1}(t)$$

Also note that

$$E_3(t^* - t) = \int_1^{\infty} \frac{\exp(-(t^* - t)x)}{x^3} dx \,;$$

$$E_2(t^* - t) = \int_1^{\infty} \frac{\exp(-(t^* - t)x)}{x^2} dx$$

Let's make the following substitutions in Eq.[8.2a, b]: x =1/m , hence $d\mu = -dx/x^2$. Thus, the integral in the first term on the right side in Eq.[8.2a] is

$$\int_0^1 \exp(-\frac{t^* - t}{m}) m dm = \int_1^\infty \exp(-(t^* - t)x) \frac{1}{x} \frac{dx}{x^2} = E_3(t^* - t)$$

and the integral in the second term on the right side in Eq.[8.2a] (which is the same as the integral in Eq.[8.2b]) is

$$\int_0^1 \exp(-\frac{t^* - t}{m}) dm = \int_1^\infty \exp(-(t^* - t)x) \frac{dx}{x^2} = E_2(t^* - t)$$

Therefore, Eq.[8.2a,b] can be written as

$$F_n^{\uparrow}(t) = 2pB_n(t^*)E_3(t^* - t) + 2p\int_t^{t^*} E_2(t' - t)B_n(t')dt' \quad [8.3a]$$

$$F_n^{\downarrow}(t) = 2p\int_0^t E_2(t - t')B_n(t')dt' \quad [8.3b]$$

- **Eq.[8.3a, b] is a final solution for infrared monochromatic fluxes** in the plane-parallel gaseous atmosphere.

NOTE: The **exponential integral E_n** cannot be analytically integrated but it can be pre-calculated using existing programs without any serious difficulties.

- **Line-by-line code** provides an “exact” solution of Eq. [8.3a, b] (recall Lecture 6)

Recall that the **total (integral) flux** is defined as $F(\tau) = \int F_\nu(\tau)d\nu$

Thus, we obtain the total (integral) upward and downward fluxes in the plane-parallel atmosphere by integrating Eq.[8.3a, b] over the entire infrared spectrum,

$$F^{\uparrow}(\tau) = 2\pi\int_0^\infty B_\nu(\tau^*)E_3(\tau^* - \tau)d\nu + 2\pi\int_0^\infty \int_\tau^{\tau^*} E_2(\tau' - \tau)B_\nu(\tau')d\tau' d\nu \qquad [8.4a]$$

$$F^{\downarrow}(\tau) = 2\pi\int_0^\infty \int_0^{\tau} E_2(\tau - \tau')B_\nu(\tau')d\tau' d\nu \qquad [8.4b]$$

Let's introduce **the transmission function** for the radiative flux (called **diffuse transmission function** or **slab transmission function**).

Monochromatic diffuse transmission function (or **transmittance):**

$$T_\nu^f(\tau) = 2\int_0^1 T_\nu(\tau/\mu)\mu d\mu \qquad [8.5]$$

where $T_\nu(\tau/\mu)$ is the monochromatic transmittance defined in Lecture 5, part 3.

NOTE: $T_\nu(\tau/\mu)$ gives the attenuation of intensity passing a layer with the optical depth τ.

Spectral diffuse transmission function (or **transmittance):**

$$T_{\bar{\nu}}^f(\tau) = 2\int_0^1 T_{\bar{\nu}}(\tau/\mu)\mu d\mu \qquad [8.6]$$

Using the definition of a monochromatic diffuse transmittance and solution of the radiative transfer equation expressed via **the transmittance** Eq.[6.4a, b], the solution for fluxes can be written as

$$F_n^{\uparrow}(t) = pB_n(t^*)T_n^f(t^* - t) - \int_t^{t^*} pB_n(t') \frac{dT_n^f(t' - t)}{dt'} dt' \qquad [8.7a]$$

and

$$F_n^{\downarrow}(t) = \int_0^t pB_n(t') \frac{dT_n^f(t - t')}{dt'} dt' \qquad [8.7b]$$

NOTE: On the right side of Eq.[8.7a] for the upward flux, the first term gives the surface emission that is attenuated to the level t and the second term gives the emission from the atmospheric layers characterized by the Planck function multiplied by the **weighting function** dT_n^f / dt . Likewise, the downward flux at a given layer (Eq.[8.7b]) is produced by the emission from the atmospheric layers.

2. Infrared radiative heating/cooling rates.

- Radiative processes may affect the dynamics and thermodynamics of the atmosphere through the generation of **radiative heating/cooling rates**.

NOTE: The thermodynamic equation for the temperature changes in the atmosphere (i.e. the first law of thermodynamic for moist air) includes **the radiative energy exchange term (i.e. total radiative heating/cooling rates** which are solar plus infrared heating/cooling rates). In this lecture we discuss IR radiative rates only (solar will be discussed later in the course).

Let's introduce the **monochromatic net flux** at a given height defined as

$$F_n(z) = F_n^{\uparrow}(z) - F_n^{\downarrow}(z)$$

Also we can define total **net flux:**

$$F(z) = F^{\uparrow}(z) - F^{\downarrow}(z)$$

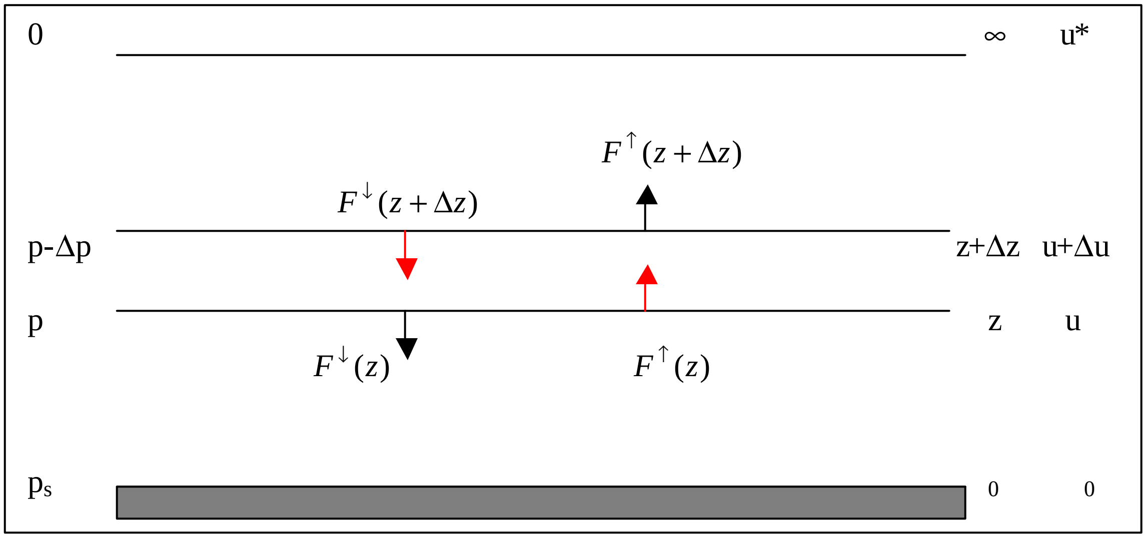
Introducing the net flux ***F*(z+Dz)** at the level **z+Dz**, we find the **net flux divergence** for the layer D**z** is

$$\Delta F = F(z + \Delta z) - F(z)$$

NOTE: The **net flux divergence** gives the radiative gain by a layer due to absorption or lost by a layer due to the emission, depending on a sign of D**F.**

***F*(z+Dz) < *F*(z) (hence DF < 0) => a layer gains radiative energy => heating**

***F*(z+Dz) > *F*(z) (hence DF > 0) => a layer losses radiative energy => cooling**



The IR **radiative heating (or cooling) rate** is defined as the rate of temperature change of the layer ***dz*** due to IR radiative energy gain (or loss):

$$\left(\frac{\partial T}{\partial t}\right)_{IR} = \int_0^{\infty}\left(\frac{\partial T}{\partial t}\right)_{n} dv = -\frac{1}{c_p r}\frac{dF(z)}{dz} \qquad [8.8]$$

where c_p is the specific heat at the constant pressure (c_p = 1004.67 J/kg/K) and r is the air density in a given layer.

Heating /cooling radiative rates may be expressed in pressure coordinates:

From hydrostatic equation (see Lecture 4) we have

$$dp = -r\, g\, dz$$

with g = 9.806 m/s^2 is gravitational acceleration.

Using $dz = -\,dp/r\, g$, we have

$$\left(\frac{\partial T}{\partial t}\right)_{IR} = \frac{g}{c_p}\frac{dF(z)}{dp} \qquad [8.9]$$

NOTE: g/c_p is the adiabatic lapse rate.

NOTE: Lab 5 is devoted to the detail modeling of the IR radiative heating rates in the gaseous atmosphere.

3. Concept of broadband flux emissivity

- The **broadband flux emissivity** approach allows calculation of infrared fluxes and heating/cooling rates utilizing the temperature in terms of the Stefan-Boltzmann law instead of the Planck function.

Based on Eq.[8.7 a, b], the total upward and downward fluxes in the path length u coordinates may be expressed as

$$F^{\uparrow}(u) = \int_0^{\infty} pB_n(T_s)T_n^f(u)dn$$

$$+\int_0^{\infty}\int_0^{u} p\,B_n(T(u'))\frac{dT_n^f(u-u')}{du'}du'dn \qquad [8.10a]$$

and

$$F^{\downarrow}(u) = \int_0^{\infty}\int_{u^*}^{u} pB_n(T(u'))\frac{dT_n^f(u'-u)}{du'}du'dn \qquad [8.10b]$$

From the Stefan-Boltzmann law (see Lecture 3), we have

$$\int_0^{\infty} pB_n(T)dn = s_B T^4$$

Let's define **the isothermal broadband emissivity** as

$$e^f(u,T) = \frac{\int_0^{\infty} pB_n(T)(1-T_n^f(u))dn}{s_B T^4} \qquad [8.11]$$

Using the **isothermal broadband emissivity,** it can be shown (see L92: 2.7.2) that Eq.[8.10a, b] may be approximated as

$$F^{\uparrow}(u) \cong s_B T_s^4 (1 - e^f(u, T_s)) - \int_0^u s_B T^4(u') \frac{de^f(u-u', T(u'))}{du} du' \quad [8.12a]$$

and

$$F^{\downarrow}(u) \cong \int_u^{u^*} s_B T^4(u') \frac{de^f(u'-u, T(u'))}{du'} du' dn \quad [8.12b]$$

NOTE: If the **isothermal broadband emissivity** is known, the broadband fluxes and heating/cooling rates can be easily calculated from Eq.[812a, b].

NOTE: There are various parameterizations of the **isothermal broadband emissivity** to account for IR absorption by atmospheric gases (e.g., see L92: 2.7.3).