

AT622 Section 1

Electromagnetic Radiation

The aim of this section is to introduce students to elementary concepts of radiometry and to describe some basic radiation laws relevant to the course.

1.1 Electromagnetic Radiation

Electromagnetic radiation from the sun is the principal source of energy that drives circulations in both the atmosphere and ocean. This radiation, in the form of a wave, is generated by oscillating (or, more generally, time varying) electric charges, which in turn generate an oscillating electric field. A characteristic of an oscillating electric field is that it produces an accompanying oscillating magnetic field that further produces an oscillating electric field. Therefore these fields, initiated by the oscillating charge, proceed outwards from the original charge, each creating the other. A visualization of such a propagating wave is given in Fig. 1.1. It was James Clerk Maxwell, who, more than a century ago, provided us with the theoretical synthesis of this phenomenon. Detailed account of this work can be found in most standard texts on electromagnetic radiation.

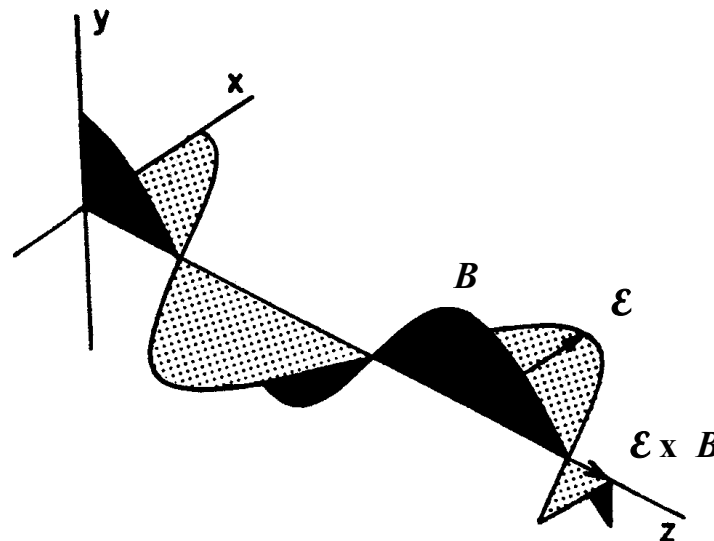


Fig. 1.1 Schematic view of a time-harmonic electromagnetic wave propagating along the z-axis. The oscillating electric E and magnetic H fields are shown. Note that the oscillations are in the x - y plane and perpendicular to the direction of propagation.

There are a number of basic properties that distinguish different electromagnetic (EM) waves. One is the rate of oscillation of the E and B fields (that is the frequency of the oscillation), the second is the amplitude of the wave (this defines the energy carried by the wave), and the third is the state of polarization of the wave. This third property is something we will not consider further in this course although it is fundamental to many topics relevant to remote sensing (and thus to AT652).

Electromagnetic theory predicts that the EM wave travels at a speed that depends on the medium through which it travels. This speed, c , can be related to the speed of propagation of light in a vacuum (namely c_0) by the formula:

$$c = \frac{c_0}{n}, \quad c_0 = 2.998 \times 10^8 \text{ ms}^{-1}$$

where n is the refractive index of the medium (more on this later). For most gases, the refractive index is close to unity especially at the wavelengths of interest to topics considered in these notes. For example, air at room temperature has $n = 1.00029$ over the visible spectrum (refer to Fig. 1.2).

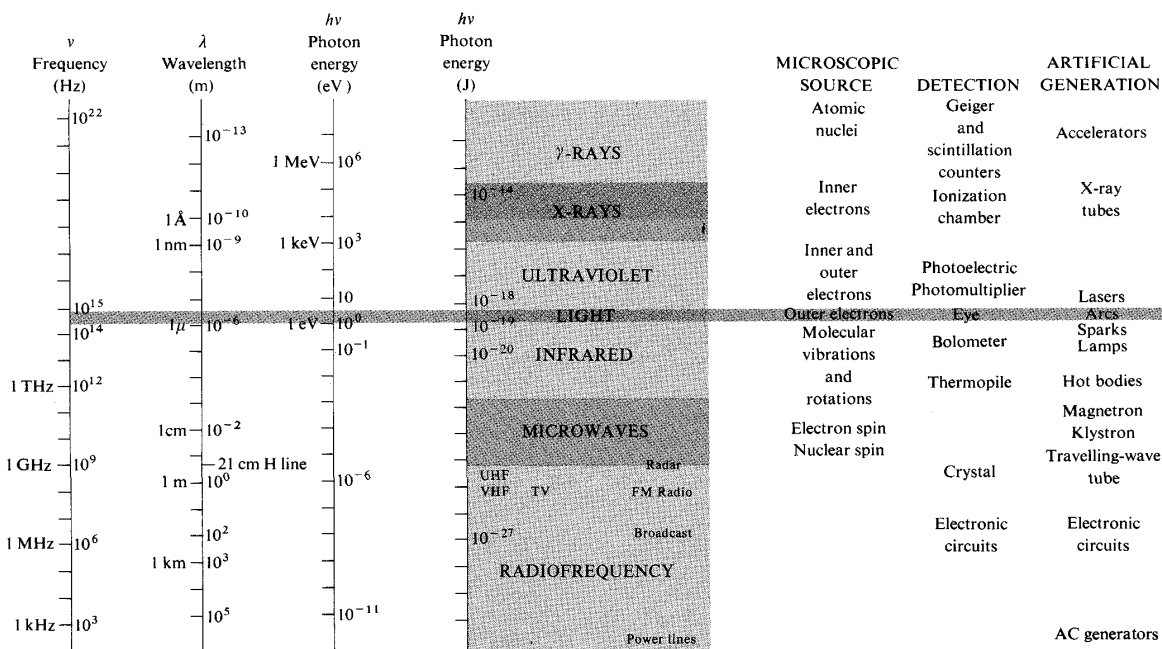


Fig 1.2. The electromagnetic-photon spectrum.

The wavelength of the EM wave depends upon the frequency of the oscillations. We shall denote this frequency (number of oscillations per second) by ν , and it is related to c by

$$\nu = \frac{c}{\lambda}, \quad (1.1)$$

where λ is the wavelength of the wave. For example, red light with a wavelength of 0.7 micrometers (μm) corresponds to a frequency of 4.3×10^{14} oscillations per second, while violet light, at 0.4 μm , corresponds to 7.5×10^{14} oscillations per second. An alternate way of describing the frequency of radiation is in terms of *wavenumber*

$$\tilde{\nu} = \frac{1}{\lambda}, \quad (1.2)$$

which is a count of the number of wave crests or troughs in a given unit of length. For example, red light has 14,286 wave crests in a centimeter whereas 25,000 crests can be counted in a centimeter of violet light. Wavenumber is the measure often used by spectroscopists and others involved in experimental measurements of the interaction of radiation with matter.

Example 1.1: What is the frequency and wavenumber of 10 μm radiation?

$$\lambda = 10 \mu\text{m} = 10 \times 10^{-6} \text{ m}$$

$$\nu = \frac{c_0}{\lambda} = \frac{3 \times 10^8 \text{ m} \cdot \text{s}^{-1}}{10 \times 10^{-6} \text{ m}} = 3 \times 10^{13} \text{ Hz}$$

$$\tilde{\nu} = \frac{1}{10 \times 10^{-6} \text{ m}} = 1000 \text{ cm}^{-1}$$

(a) *Electromagnetic Spectrum*

Even before Maxwell, the spectrum of electromagnetic radiation (that is the range of wavelengths or frequencies of the radiation) was known to extend beyond the visible (i.e., beyond those wavelengths detectable by the human eye). In fact, we now know that the visible portion of the spectrum, from 0.4 μm to 0.7 μm , is just a small part of a much broader spectrum of electromagnetic radiation. The character of the radiation and the way it interacts with matter is vastly different depending on the wavelength of the radiation. The radiation that is relevant to this course is shown in Fig. 1.3

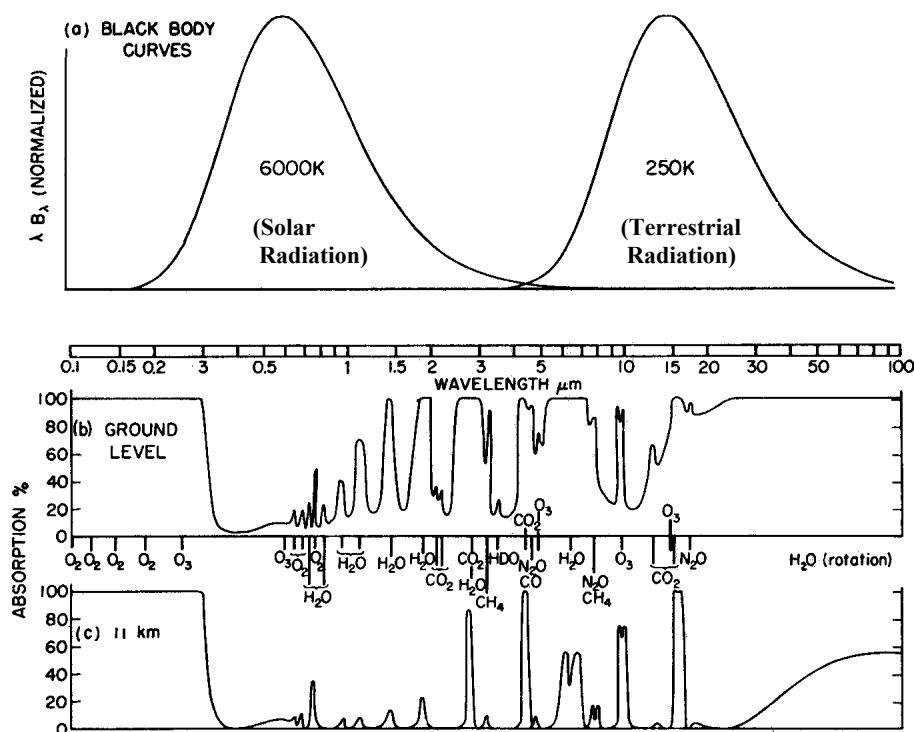


Fig. 1.3 Atmospheric absorptions. (a) Blackbody curves for 6000 K and 250 K. (b) Atmospheric absorption spectrum for a solar beam reaching ground level. (c) The same for a beam reaching the temperate tropopause. The axes are chosen so that areas in (a) are proportional to radiant energy. Integrated over the earth's surface and over all solid angles, the solar and terrestrial fluxes are equal to each other; consequently, the two blackbody curves are drawn with equal areas. Conditions are typical of mid-latitudes and for a solar elevation of 40° or for a diffuse stream of terrestrial radiation.

In the solar/terrestrial spectrum shown in Fig. 1.3, the frequency domains of greatest interest are the ultraviolet, visible, and infrared wavelengths. The ultraviolet frequencies range from the extreme ultraviolet (EUV) at 10 nm to the near ultraviolet (NUV) at 400 nm as shown.

Extreme Ultraviolet – EUV. The extreme ultraviolet, sometimes abbreviated XUV, is defined here as 10 nm to 100 nm. The division between the EUV and the FUV is frequently considered to be the ionization threshold for molecular oxygen at 102.8 nm. The EUV solar radiation is responsible for photoionization at ionospheric altitudes. The division between the EUV and the x-ray regions corresponds very roughly to the relative importance of interactions of the photons with valence shell and inner shell electrons, respectively.

Far Ultraviolet – FUV. This region extends from about the beginning of strong oxygen absorption to about the limit of availability of rugged window materials, the lithium fluoride transmission limit. The range as used here extends from 100 to 200 nm.

Middle Ultraviolet – MUV. The middle UV covers the region from 200 to 300 nm, which is approximately the region between the solar short wavelength limit at ground level and the onset of strong molecular oxygen absorption. Most solar radiation in this range is absorbed in the atmosphere by ozone.

Near Ultraviolet – NUV. This region covers wavelengths from 300-400 nm, and represent roughly the limits between the solar ultraviolet that reaches the surface of the earth and the limit of human vision in the visible.

The biomedical community uses a different convention:

UV-C	15-280 nm
UV-B	280-315 nm
UV-A	315-400 nm

UV-C is absorbed entirely in the upper atmosphere and is of practical significance to the biomedical community only in the sense that it is frequently used to sterilize surfaces and kill bacteria. UV-B is responsible for Vitamin-D production by the skin. Both UV-A and UV-B activate melanin in the skin, which is responsible for the darker appearance after exposure to the sun.

The infrared spectrum is occasionally divided into the near and far infrared.

Near Infrared. The portion of the spectrum beyond the visible (0.7 μm) where the amount of solar radiation is significant. Generally defined in the 0.7 μm to 2.5 μm range.

Far Infrared. The portion of the spectrum beyond near infrared where the Earth's radiation dominates. 2.5 μm to 1000 μm range.

Microwaves, while not contributing to the energy incident upon Earth or emitted from it, play an important role in remote sensing as well as communications. Figure 1.4 shows the allocation of the microwave spectrum to different users.

(b) *The Mathematical Form of an EM Wave*

The oscillatory E and H fields can be presented in a classical way as a harmonic oscillator of the form

$$E = E_0 \cos k(x - ct) \quad (1.3a)$$

where the quantity E_0 is the amplitude of the wave and, as we shall see later, the energy carried by the wave is related to the square of this amplitude and is the property of most interest and relevance to atmospheric sciences. The quantity $k (= 2\pi\tilde{\nu})$ is also referred to as wavenumber, but this should not prove to be a source of confusion as $\tilde{\nu}$ and k are used in different contexts; k generally applies to wave propagation, whereas $\tilde{\nu}$, is used, as in the previous section, to discriminate regions of the electromagnetic spectrum. Equation. (1.3a) can also be written in the form

$$E = E_0 \cos(kx - \omega t) \quad (1.3b)$$

where $\omega = kc = 2\pi c/\lambda$ is the angular frequency of the wave and, according to Eqn. (1.1), $\omega = 2\pi\nu$.

The argument of the cosine function in Eqn. (1.3a) also has a particular meaning. It is represented by the function ϕ

$$\phi = k(x - ct) \quad (1.4)$$

and is referred to as the *phase* of the wave. A tidier expression for a harmonic oscillator is

$$E(x,t) = E_0 e^{ik(x-ct)}, \quad (1.5)$$

where it may be taken for granted that the real part of this expression represents the wave.

The general representation of the harmonic wave requires that the displacement E be specified at $x = 0$ and $t = 0$. We specify this initial displacement (initial since it is defined at $t = 0$) in terms of a constant phase, ϕ_0 , at $x = 0$ and $t = 0$. For this general case,

$$\phi = \phi_0 + k(x-ct) \quad (1.6)$$

and

$$E(x,t) = E_0 e^{i\phi}. \quad (1.7)$$

Simple algebraic manipulations show that the square of the wave amplitude, given as

$$|E(x,t)|^2 = |E_0|^2, \quad (1.8)$$

is the same for all x and t since E_0 is a constant. The energy transferred by the wave, related to $|E_0|^2$, does not vary along its path of propagation and is independent of our definition of ϕ_0 . It is only the interaction of the wave with matter that alters the energy of the wave as it propagates. These interactions and the potential they offer for modulating the atmosphere is of interest to the atmospheric scientist.

Fig. 1.4 Allocation of the microwave frequency spectrum. Details are available at: <http://www.ntia.doc.gov/osmhome/allochrt.html>.

1.2 Energy Carried by an EM Wave

An electromagnetic wave, traveling through space at the speed of light, carries electromagnetic energy, which is detected by sensors that respond to this energy. In the next section, we will discuss in a more geometrical way how we describe the flow of energy carried by an EM wave. But for now we will endeavor to see what defines this energy. Energy flows in the direction in which the wave advances and this direction of propagation is defined by the vector cross product of the electric and magnetic fields, $\vec{E} \times \vec{B}$. The energy per unit area per unit time flowing perpendicular into a surface in free space is given by the *Poynting vector* \vec{S} , where

$$\vec{S} = c^2 \epsilon_0 \vec{E} \times \vec{B} \quad (1.9)$$

c is the speed of light and ϵ_0 is the vacuum permittivity ($\epsilon_0 = 8.85 \times 10^{-12} \text{ F}\cdot\text{m}^{-1}$). Energy per unit time is power, so the SI units of \vec{S} are Wm^{-2} . At the frequencies of interest to the topics of this class, the fields \vec{E} , \vec{B} , and \vec{S} oscillate at rapid rates. It thus remains impractical to measure instantaneous values of \vec{S} directly. Rather, we measure its average magnitude, $\langle S \rangle$, over some time interval that is a characteristic of the detector. This time averaged quantity is referred to as the radiant-flux density. Strictly speaking, the flux density emerging from the surface is known as the *exitance* and the flux density incident on the surface is called the *irradiance*. To avoid unnecessary complications with nomenclature, we refer to the flux density onto or from a surface as either irradiance or flux and use the symbol F to represent this quantity.

When the flow of light is nonparallel and when the detector collects the light confined to a range of directions, specified by a small element of solid angle $d\Omega$, then the quantity sensed is the *intensity*, defined as $\langle S \rangle / d\Omega$ and has units of $\text{Wm}^{-2}\text{ster}^{-1}$. This quantity, referred to as a radiance, is used throughout and we will denote it by the symbol I (more about this below).

We can consider a more direct relationship between the energy carried by an electromagnetic wave and the amplitudes of the electric and magnetic fields by considering the simple case of a plane wave of the form

$$\vec{E} = \vec{E}_0 \cos(kx - \omega t). \quad (1.10)$$

The magnetic field also has the form $\vec{B} = \vec{B}_0 \cos(kx - \omega t)$ and therefore

$$S = c^2 \epsilon_0 \vec{E} \times \vec{B}_0 = c^2 \epsilon_0 \vec{E}_0 \times \vec{B}_0 \cos^2(kx - \omega t). \quad (1.11)$$

Hence

$$\langle S \rangle = c^2 \epsilon_0 |\vec{E}_0 \times \vec{B}_0| \langle \cos^2(kx - \omega t) \rangle, \quad (1.12)$$

and the time average is calculated for an interval of length T according to

$$\langle \cos^2(kx - \omega t) \rangle = \frac{1}{T} \int_t^{t+T} \cos^2(kx - \omega t') dt' = \frac{1}{2} - \frac{1}{4\omega T} [\sin 2(kx - \omega(t+T)) - \sin 2(kx - \omega t)]. \quad (1.13)$$

When $T \gg t$, $\omega T \gg 1$ and $\langle \cos^2(kx - \omega t) \rangle \rightarrow 1/2$. Since $E_0 = cB_0$,

$$F = \langle S \rangle = \frac{c\epsilon_0}{2} E_0^2 \quad (1.14)$$

or

$$F = c\epsilon_0 \langle E^2 \rangle, \quad (1.15)$$

where $\langle E^2 \rangle = E_0^2/2$. It also follows that $I = \langle S \rangle / d\Omega$.

Example 1.2: Consider the following problem: a plane, sinusoidal, linearly polarized electromagnetic wave of wavelength $\lambda = 5.0 \times 10^{-7} \mu\text{m}$ travels in a vacuum along the x axis. The average flux of the wave per unit area is 0.1 Wm^{-2} and the plane of vibration of the electric field is parallel to the y axis. Write the equations describing the electric and magnetic fields of the wave.

The solution is as follows: The wavenumber is $k = 2\pi/\lambda = 4 \times 10^6 \text{ m}^{-1}$. Given the following,

$$c\epsilon_0 = \frac{10^7}{4\pi c}$$

then the amplitude

$$E_0 = \frac{2 \times 0.1}{c\epsilon_0} = \sqrt{24\pi}$$

and the form of the E wave is

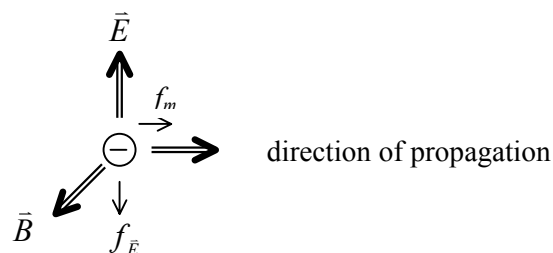
$$E_y(t) = \sqrt{24\pi} \cos 4\pi \times 10^6 (x - ct) \text{ [NC}^{-1}\text{]}$$

and the magnetic field is governed by

$$B(t) = \frac{E_y(t)}{c} \text{ [T]}.$$

1.3 Momentum and Radiation Pressure

An electromagnetic wave, aside from carrying energy, also carries momentum. If an electromagnetic wave is absorbed or reflected by a material, it will impart momentum to the electrons in the material, which is subsequently transmitted to the lattice structure as a whole.



The electric field exerts a force $\vec{f}_E = q\vec{E}$, which drives an electron with a velocity \vec{v}_e . (Note that the symbol \vec{f} is used instead of F to distinguish it from the radiative flux F used throughout these notes). The magnetic field, in turn, exerts a force $f_m = q\vec{v}_e \times \vec{B}$. Although f_E changes its direction as \vec{E} varies, having the direction opposite to \vec{E} at all times ($q < 0$), f_m is always in the direction of propagation since \vec{v}_e and \vec{B} reverse directions simultaneously.

Thus the electron undergoes rapid oscillations in the direction of the \vec{E} field, and a small increase in speed in the direction in which the light propagates. Because the time average of the transverse oscillations is zero, the net force on the electron is given by

$$\langle f \rangle = q \langle \vec{v}_e \times \vec{B} \rangle = qv_e B \hat{i} = \frac{q}{c} \langle v_e E \rangle \quad (1.16)$$

Power is the rate at which work is done $\frac{dW}{dt}$ and can be expressed (assuming constant forces) as

$$\vec{P} = \vec{v} \cdot \vec{f} \quad (1.17)$$

$$\vec{P} = \vec{v}_e \cdot (\vec{f}_E + \vec{f}_m) \quad (1.18)$$

$$= \vec{v}_e \cdot (q\vec{E} + \vec{v}_e \times \vec{B}) \quad (1.19)$$

$$= q \langle v_e E \rangle \quad (1.20)$$

Combining Eqn. (1.16) for the force, or the equivalently, for the rate of change of momentum with Eqn. (1.20), one obtains

$$\langle f \rangle = \frac{P}{c} = \frac{\langle S \rangle \cdot \text{Area}}{c} \quad (1.21)$$

Finally, the radiation pressure, \mathbf{P} , is given by $\frac{f}{\text{Area}}$

$$\mathbf{P} = \frac{\langle f \rangle}{\text{Area}} = \frac{\langle S \rangle}{c} \quad (1.22)$$

Equation (1.22) is appropriate if radiation is absorbed by the material. If a photon is perfectly reflected, the change in momentum is twice that computed here, and $\mathbf{P} = 2\langle S \rangle/c$.

1.4 Problems

Problem 1.1

The wavelength of the radiation absorbed during a particular spectroscopic transition is observed to be $10\ \mu\text{m}$. Express this in frequency (Hz) and in wavenumber (cm^{-1}), and calculate the energy change during the transition in both joules per molecule and joules per mole. If the energy change were twice as large, what would be the wavelength of the corresponding radiation?

Problem 1.2

Electromagnetic radiation from the sun falls on top of the Earth's atmosphere at the rate of $1.37 \times 10^3\ \text{Wm}^{-2}$. Assuming this is to be plane wave radiation, estimate the magnitude of the electric and magnetic field amplitudes of the wave. The units of the electric field are $\text{m kg s}^{-2}\text{Co}^{-1}$ and the units of the magnetic field are $\text{kg s}^{-1}\text{Co}^{-1}$, which is also known as a telsa (T).

Problem 1.3

Assume that a 100 W lamp of 80% efficiency radiates all its energy isotropically. Compute the amplitude of both the electric and magnetic fields 2 m from the lamp.

Problem 1.4

The average power of a broadcasting station is $10^5\ \text{W}$. Assume that the power is radiated uniformly over any hemisphere concentric with the station. For a point 10 km from the source, find the magnitude of the Poynting vector and the amplitudes of the electric and magnetic fields. Assume that at that distance the wave is plane.

Problem 1.5

A radar transmitter emits its energy within a cone having a solid angle of 10^{-2} sterad. At a distance of $10^3\ \text{m}$ from the transmitter the electric field has an amplitude of $10\ \text{Vm}^{-1}$. Find the amplitude of the magnetic field and the power of the transmitter.

Problem 1.6

Assume that a 100-W lamp of 80% efficiency radiates all its energy isotropically. Compute the amplitude of the electric and magnetic fields 2 m from the lamp.

Problem 1.7

Radio waves received by a radio set have an electric field of maximum amplitude equal to $10^{-1}\ \text{Vm}^{-1}$. Assuming that the wave can be considered as plane, calculate: (a) the amplitude of the magnetic field, (b) the average intensity of the wave, and (c) the average energy density.

Assuming that the radio set is 1 km from the broadcasting station and that the station radiates energy isotropically, determine the power of the station.