Lecture 10

Principles of passive remote sensing using extinction and scattering: Scattering as a source of radiation. Multiple scattering. The two-stream approximation.

Objectives:

1. Scattering as a source of radiation.

2. Two-stream approximations.

3. Examples of passive remote sensing using extinction and scattering in the solar spectral region.

Required Reading:

G: 6.3, 6.4, 6.6

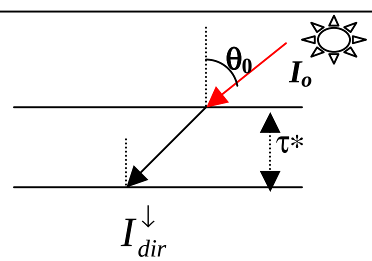
1.Scattering as a source of radiation.

- The solar radiation field is traditionally considered as a sum of two distinctly different components: **direct** and **diffuse**: $I = I_{dir} + I_{dif}$

Direct solar radiation is a part of solar radiation filed that has survived the extinction passing a layer with optical depth τ^* and it obeys the Beer-Bouguer-Lambert law (or Extinction law):

$$I_{dir}^{\downarrow} = I_0 \exp(-\tau^* / \mu_0) \qquad [10.1]$$

where I_0 is the solar intensity at a given wavelengths at the top of the atmosphere and μ_0 is a cosine of the solar zenith angle θ_0 ($\mu_0 = cos(\theta_0)$).



Since the solid angle of the sun is small, Eq. [10.1] can be re-written as

$$F_{dir}^{\downarrow} = F_0 \exp(-\tau^* / \mu_0)$$

Measurements of direct solar radiation are used for

1) retrieval of aerosol optical depth (based on the Langley method) and aerosol particle size distribution (see Computer Lab 5 and Homework 2)
2) retrieval of ozone (Lecture 11) and other absorbing gases
3) determination of the solar constant:

   Smithsonian long method:

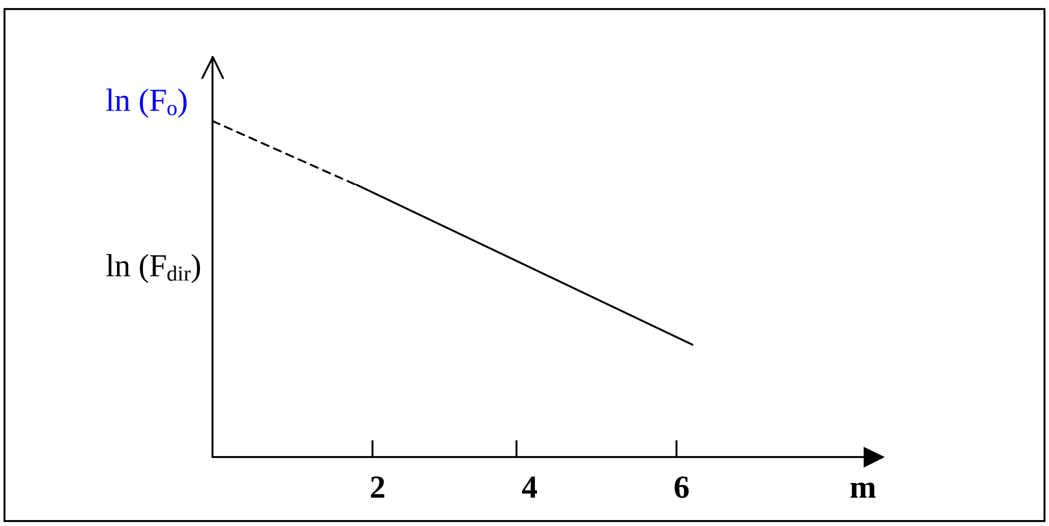
   Assume that the optical depth remains constant for the range of solar zenith angle

   $$\ln(F_{dir}^{\downarrow}) = \ln F_0 + mT$$

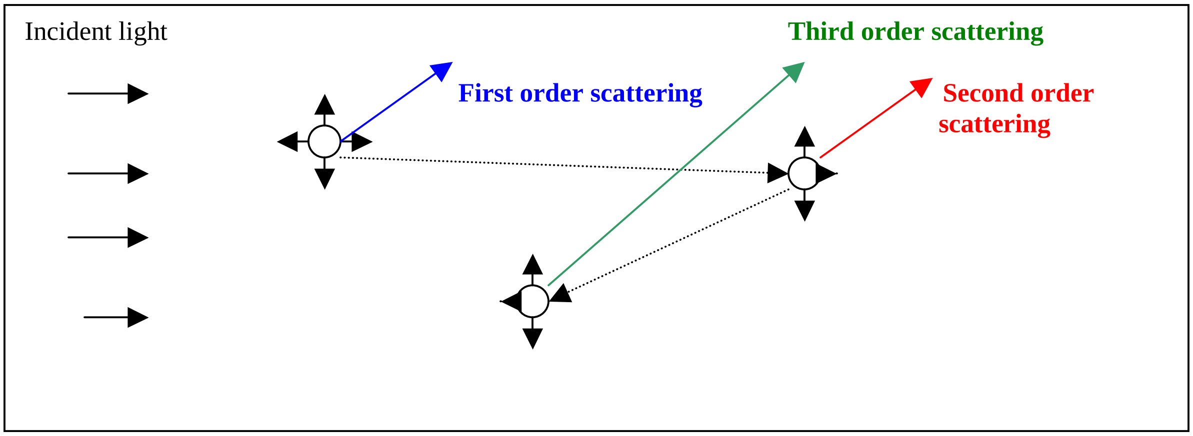
   where T is transmission at the measured wavelength;

   m is the air mass: ***m***=1/μ_0

NOTE: DO NOT CONFUSE ***m*** and METEOROLOGICAL AIR MASS



Diffuse radiation arises from the light that undergoes one scattering event (**single scattering**) or many (**multiple scattering**).



Scattering is a source of light at a given direction because light is scattered into this beam from other directions.

For single scattering

$$\delta I_\lambda(\vec{\Omega}) = k_s ds \frac{P(\vec{\Omega},\vec{\Omega}')}{4\pi} I_\lambda(\vec{\Omega}')\delta\Omega'$$

where $I_\lambda(\vec{\Omega}')$ is incident intensity in the direction $\vec{\Omega}'(\mu',\phi')$.

For **multiple scattering,** integrating over all directions:

$$dI_\lambda(\vec{\Omega}) = k_s ds \int_{4\pi} \frac{P(\vec{\Omega},\vec{\Omega}')}{4\pi} I_\lambda(\vec{\Omega}')d\Omega'$$

NOTE: The above equation shows that the phase function redirects the incident intensity in the direction $\vec{\Omega}'(\mu',\phi')$ to the direction $\vec{\Omega}(\mu,\phi)$, and the integral account for all possible scattering events within the 4π solid angle.

According to the Beer-Bouguer-Lambert law, scattering radiance from path ***ds*** is

$$dI_{\lambda} = k_{e,\lambda} J_{\lambda} ds \quad \text{(see lecture 9)}$$

thus **the scattering source function** is

$$J_{\lambda}(\vec{\Omega}) = \frac{\omega_{0,\lambda}}{4\pi} \int_{\vec{\Omega}'} I(\vec{\Omega}') P(\vec{\Omega}, \vec{\Omega}') d\Omega' \qquad [10.2]$$

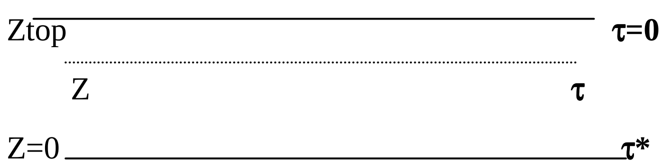
<u>The scattering source function:</u>

1) has units of intensity

2) plays the role of the Planck function in thermal radiative transfer but the scattering source function is more complex

3) depends on the radiation intensity in the incident direction, $I_{\lambda}(\vec{\Omega}')$; fraction of radiation which is scattered, $\omega_{0,\lambda}$; fraction scattered into the new direction $\frac{P(\vec{\Omega}, \vec{\Omega}')}{4\pi} d\Omega'$

Consider a plane-parallel atmosphere



The monochromatic radiative transfer equation for a plane-parallel atmosphere expresses the net change in intensity due to extinction and scattering along path ***dz:***

dI = dI (extinction) + dI(scattering)

$$\mu \frac{dI_{\lambda}(z, \mu, \varphi)}{dz} = -k_e [I_{\lambda}(z, \mu, \varphi) - J_{\lambda}(z, \mu, \varphi)] \qquad [10.3]$$

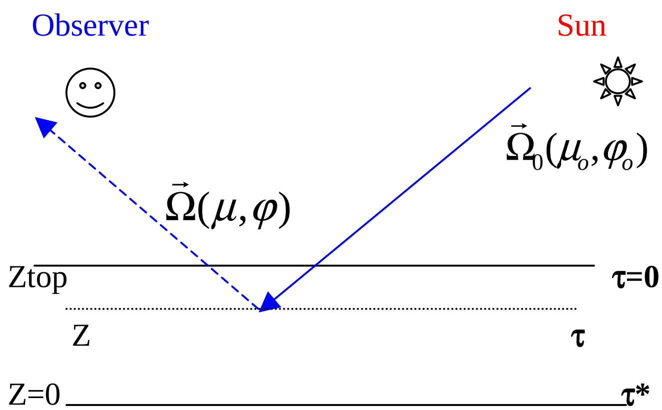
or using the optical depth as the vertical coordinate

$$\mu \frac{dI_{\lambda}(\tau; \mu; \varphi)}{d\tau} = I_{\lambda}(\tau; \mu, \varphi) - J_{\lambda}(\tau, \mu, \varphi) \qquad [10.4]$$

In solving the radiative transfer equation in the atmosphere, the **collimated distribution** is a commonly used approximation for the intensity of an incident solar beam at the top of the atmosphere, in which the finite size of the sun is ignored:

$$I_0 = F_0 \delta(\vec{\Omega} - \vec{\Omega}_0)$$

where $\mathbf{F_0}$ is solar flux and $\delta(\vec{\Omega} - \vec{\Omega}_0)$ is the Dirac δ-function (here it has units of inverse solid angle). The above equation is physically meaningful only when it is integrated over some finite solid angle.



First-order scattering :

For the direct beam from the Beer-Bouguer-Lambert law, we have

$$F^{\downarrow}(z) = F_0 \exp(-k_e(z_t - z)/\mu_0) = F_o \exp(-\tau/\mu_0)$$

where $\boldsymbol{F_0}$ is the solar constant at the top of the atmosphere.

Scattering of the direct beam is the source of diffuse radiation

$$J(\tau, \mu, \varphi) = \frac{\omega_0}{4\pi} F_0 \exp(-\tau/\mu_0) P(\mu, \varphi, \mu_0, \varphi_0)$$

Thus, upwelling intensity at the level Z (or τ) can be found as

$$I^{\uparrow}(\tau, \mu, \varphi) = \int_{\tau}^{\tau^*} J(\tau', \mu, \varphi) \exp[-(\tau' - \tau)/\mu] d\tau'/\mu$$

Assuming no surface reflection (dark surface) and substituting in the source function

$$I^{\uparrow}(\tau,\mu,\varphi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \int_{\tau}^{\tau*} \exp[-(\tau' - \tau)/\mu - \tau'/\mu_0] d\tau'/\mu$$

An observer (i.e., a satellite detector) at Ztop (or $\tau = 0$) measures

$$I^{\uparrow}(0,\mu,\varphi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \frac{\mu_0}{\mu + \mu_0} [1 - \exp(-(\frac{1}{\mu_o} + \frac{1}{\mu})\tau^*)] \qquad [10.5]$$

If **$\tau^* \ll 1$ (called the single scattering approximation), Eq.10.5 simplifies to**

$$I^{\uparrow}(0,\mu,\varphi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \frac{\tau^*}{\mu} \qquad [10.6]$$

NOTE: AVHRR aerosol retrieval algorithm is based on the single-scattering approximation (see G: 6.4.2)

In the general case of multiple scattering

$$J(\tau,\mu,\varphi) = \frac{\omega_0}{4\pi} \int_0^{2\pi} \int_{-1}^{1} I(\tau,\mu',\varphi') P(\mu,\varphi,\mu',\varphi') d\mu' d\varphi' + \frac{\omega_0}{4\pi} F_0 P(\mu,\varphi,\mu_0,\varphi_0) \exp(-\tau/\mu_0) \qquad [10.7]$$

Using the source function for scattering, we can write the **radiative transfer equation for diffuse radiation** as an integro-differential equation.

$$\mu \frac{dI(\tau,\vec{\Omega})}{d\tau} = I(\tau,\vec{\Omega}) - \frac{\varpi_0}{4\pi} \int_{4\pi} I(\tau,\vec{\Omega}) P(\vec{\Omega},\vec{\Omega}') d\Omega' - \frac{\omega_0}{4\pi} F_0 P(\vec{\Omega},\vec{\Omega}_0) \exp(-\tau/\mu_0)$$

[10.8]

NOTE: To solve Eq.[10.8], one needs to know the scattering coefficient $\mathbf{k_{s,\lambda}}$, absorption coefficient $\mathbf{k_{a,\lambda}}$ and scattering phase function $\boldsymbol{P(\mu, \phi, \mu', \phi')}$ as a function of wavelength in each atmospheric layer.

- **To find a solution of the radiative transfer equation for diffuse radiation** (i.e., to solve Eq.[10.8]) , various approximate and "exact" techniques have been developed:

  Approximate methods:

  i) Single scattering approximations (see above)

  ii) Two-stream approximations (see below)

  iii) Eddington and Delta- Eddington approximations

  "Exact" methods:

  i) Discrete-ordinate technique

  ii) Adding-doubling technique

  iii) Monte-Carlo technique

2. Two-stream approximations.

Underlying idea:

Because radiative flux (or irradiance) is the angular-averaged property, one can expect that details of the angular variation of intensity are not very important for the predictions of irradiance.

Strategy:

Introduce an "effective" angular averaged intensity/flux (stream) so the integro-differential equation of radiative transfer (e.g., Eq.[10.8]) reduces to two coupled ordinary differential equations for $I^{\uparrow}$ (or $F^{\uparrow}$) and $I^{\downarrow}$ (or $F^{\downarrow}$).

Disadvantages of the two-stream approximations:

Two-steam methods provide acceptable accuracy but over a restricted range of the parameters. There is no a priory method to estimate the accuracy, so one needs to use the "exact" method to obtain an accurate solution which can be used to estimate the accuracy of two-stream solutions.

Advantages of the two-stream approximations:

Two-stream approximations are computationally efficient (therefore they are often used in climate models). They are also used in remote sensing: (i) interpretation of measured hemispherical fluxes (Computer Lab 11: Earth radiation budget) and (2) interpretation of solar reflection measurements from clouds (Homework 3).

Reflection function of an atmospheric layer is defined as

$$R(\mu,\varphi,\mu_0,\varphi_0) = I^{\uparrow}(0,\mu,\varphi)/\mu_0 F_0 \qquad [10.9]$$

Transmission function of an atmospheric layer is defined as

$$T(\mu,\varphi,\mu_0,\varphi_0) = I^{\downarrow}(\tau^*,\mu,\varphi)/\mu_0 F_0 \qquad [10.10]$$

NOTE: Eq.[10.10] uses the **<u>diffuse</u>** intensity, therefore $T(\mu,\varphi,\mu_0,\varphi_0)$ is also called the diffuse transmission function.

Planetary albedo (or local albedo or reflection) is associated with the reflected (upward) flux and defined as

$$r(\mu_0) = \frac{F_{dif}^{\uparrow}(0)}{\mu_0 F_0} = \int_0^{2\pi}\int_0^1 R(\mu,\varphi,\mu_0,\varphi_0)\mu d\mu d\varphi \qquad [10.11]$$

Diffuse transmission is associated with transmitted (downward) flux and defined as

$$t(\mu_0) = \frac{F_{dif}^{\downarrow}(\tau^*)}{\mu_0 F_0} = \int_0^{2\pi}\int_0^1 T(\mu,\varphi,\mu_0,\varphi_0)\mu d\mu d\varphi \qquad [10.12]$$

3. Examples of passive remote sensing using extinction and scattering in the solar spectral region:

- Retrieval of ozone (Lecture 11)
- Retrieval of the aerosol optical depth and particle size distribution (Lecture 10, Computer Lab 5 and Homework 2)
- Retrieval of stratospheric aerosols (based on limb viewing remote sensing) (Homework 2)
- Retrieval of ocean color (Lecture 13)
- Retrieval of cloud properties (Homework 3)
- Retrieval of surface properties (e.g., surface albedo; NDVI (Normalized Differential Vegetation Index) after the atmospheric correction is applied, see G 6.4.1)