Lecture 14

Principles of passive remote sensing using emission: Radiative transfer with emission. Measurements of path-integrated quantities.

Objectives:

1. Radiative transfer with emission.
2. Microwave radiative transfer.
3. Measurements of path-integrated quantities (precipitable water vapor, cloud liquid water).

Required reading:

G: 7.1, 7.3.1, 7.3.2

1. Radiative transfer with emission.

Atmosphere and surfaces emit infrared and microwave radiation.

According to the Kirchhoff's law (see Lecture 4): emission=absorption

Recall the Beer-Bouguer-Lambert law (Eq.[9.1], Lecture 9) for emission

$$dI_\lambda = k_{e,\lambda} J_\lambda ds$$

where $\boldsymbol{k_{e,\lambda}}$ is the volume extinction coefficient along path ***ds***.

- For a non-scattering medium in LTE, the Planck function gives the source function

$$\boxed{J_\lambda = B_\lambda} \qquad [14.1]$$

Neglecting scattering => ***volume extinction coefficient = volume absorption coefficient***

NOTE: In Lecture 7, several kinds of the absorption coefficient, $\boldsymbol{k_\lambda}$, by gases were introduced, depending on the expression of the amount of a gas.

Thus, the net change of radiation along path ***ds*** is due to the combination of emission and extinction

dI = dI (extinction) + dI(emission)

and the radiative transfer equation in the thermal region is

$$dI_\lambda = -k_\lambda I_\lambda ds + k_\lambda B_\lambda ds \qquad [14.2]$$

or

$$\frac{dI_\lambda}{ds} = -k_\lambda [I_\lambda - B_\lambda] \qquad [14.3]$$

NOTE: Eqs.[14.2]-[14.3] are often called the differential forms of the radiative transfer.

Recall that by definition $d\tau_\lambda = -k_\lambda (s) ds$

Let's re-arrange terms in Eq.[14.3] and multiply both sides by ***exp(-τ_λ)***

$$-\frac{\exp(-\tau_\lambda) dI_\lambda}{d\tau_\lambda} + \exp(-\tau_\lambda) I_\lambda = \exp(-\tau_\lambda) B_\lambda$$

and (using that ***d[I(x)exp(-x)]=exp(-x)dI(x)-exp(-x)I(x)dx***) we have

$$-d\left[I_\lambda \exp(-\tau_\lambda)\right] = \exp(-\tau_\lambda) B_\lambda d\tau_\lambda$$

Integrating the above equation along a path extending from some point s' to the end point s", it becomes

$$I_\lambda(s'') e^{-\tau_\lambda(s'')} - I_\lambda(s') e^{-\tau_\lambda(s')} = \int_{\tau(s'')}^{\tau(s')} B_\lambda(s) e^{-\tau_\lambda(s)} d\tau(s)$$

and, re-arranging terms, we have **the solution** of the radiative transfer in IR

$$I_\lambda(s'') = I_\lambda(s')e^{-[\tau_\lambda(s')-\tau(s'')]} + \int_{\tau(s'')}^{\tau(s')} B_\lambda(s)e^{-[\tau_\lambda(s)-\tau(s'')]}d\tau(s) \quad [14.4]$$

contribution from radiation incident at s' and transmitted to s''

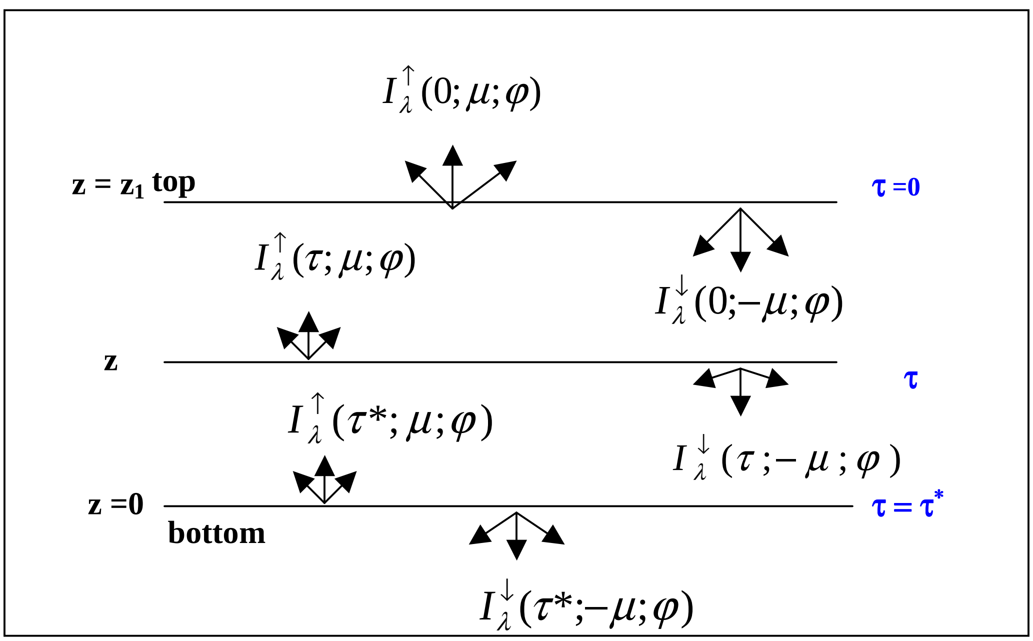
contribution from radiation emitted along the path and transmitted to s''

Let's consider a plane-parallel atmosphere ***(dz=μ ds*** and ***τ(z) = μ τ(s))***

Upward intensity $I_\lambda^\uparrow$ is for $1 \geq \mu \geq 0$ (or $0 \leq \theta \leq \pi/2$);

Downward intensity $I_\lambda^\downarrow$ is for $-1 \leq \mu \leq 0$ (or $\pi/2 \leq \theta \leq \pi$)

(using that ***cos***(0)=1; ***cos***(π/2)=0 and ***cos***(π) =-1)



NOTE: For downward intensity, μ is replaced by $-\mu$.

Eq.[14.4] gives both a upward intensity in the plane-parallel atmosphere and

$$I_\lambda^{\uparrow}(\tau;\mu;\varphi) = I_\lambda^{\uparrow}(\tau^*;\mu;\varphi)\exp\left(-\frac{\tau^*-\tau}{\mu}\right)$$

$$+\frac{1}{\mu}\int_{\tau}^{\tau^*}\exp\left(-\frac{\tau'-\tau}{\mu}\right)\ B_\lambda(\tau')d\tau' \qquad [14.5]$$

a downward intensity in the plane-parallel atmosphere:

$$I_\lambda^{\downarrow}(\tau;-\mu;\varphi) = I_\lambda^{\downarrow}(0;-\mu;\varphi)\exp\left(-\frac{\tau}{\mu}\right)$$

$$+\frac{1}{\mu}\int_{0}^{\tau}\exp\left(-\frac{\tau-\tau'}{\mu}\right)\ B_\lambda(\tau')d\tau' \qquad [14.6]$$

- In the atmospheric conditions for IR radiation, one can consider that at the surface

$$I_\lambda^{\uparrow}(\tau^*;\mu;\varphi) = B_\lambda(T_s) \text{ or } I_\lambda^{\uparrow}(\tau^*;\mu;\varphi) = \varepsilon_\lambda B_\lambda(T_s)$$

  no thermal incident radiation at the TOA

$$I_\lambda^{\downarrow}(0;\mu;\varphi) = 0$$

  no dependence on azimuthal angle, φ.

Thus Eqs.[14.5] and [14.6] can be re-written in the wavenumber domain as

$$I_\nu^{\uparrow}(\tau;\mu) = B_\nu(\tau^*)\exp\left(-\frac{\tau^*-\tau}{\mu}\right)$$

$$+\frac{1}{\mu}\int_{\tau}^{\tau^*}\exp\left(-\frac{\tau'-\tau}{\mu}\right)\ B_\nu(\tau')d\tau' \qquad [14.7]$$

$$I_\nu^{\downarrow}(\tau;-\mu) = \frac{1}{\mu}\int_0^{\tau} \exp(-\frac{\tau-\tau'}{\mu})\ B_\nu(\tau')d\tau' \qquad [14.8]$$

Eqs.[14.7] - [14.8] can be expressed in terns of monochromatic transmittance.

Recall that

$$T_\nu(\tau/\mu) = \exp(-\frac{\tau_\nu}{\mu}) \qquad [14.9]$$

and the differential form is

$$\frac{dT_\nu(\tau/\mu)}{d\tau} = -\frac{1}{\mu}\exp(-\frac{\tau_\nu}{\mu}) \qquad [14.10]$$

- **Multiplication law of transmittance** states that when several gases absorb, the **monochromatic transmittance** is a product of the monochromatic transmittances of individual gases:

$$T_{\nu,1,2...N} = T_{\nu,1}T_{\nu,2}...T_{\nu,N}$$

Thus the formal solutions for monochromatic upward and downward **in terms of transmittance are:**

$$I_\nu^{\uparrow}(\tau;\mu) = B_\nu(\tau^*)T_\nu((\tau^*-\tau)/\mu)$$

$$-\int_{\tau}^{\tau^*} B_\nu(\tau')\frac{dT_\nu((\tau'-\tau)/\mu)}{d\tau'}d\tau' \qquad [14.11]$$

$$I_\nu^{\downarrow}(\tau;-\mu) = \int_0^{\tau} B_\nu(\tau')\frac{dT_\nu((\tau-\tau')/\mu)}{d\tau'}d\tau' \qquad [14.12]$$

2. Microwave radiative transfer.

According to the Rayleigh-Jeans distribution (see Lecture 4, Eqs.[4.4a,b]):

Brightness temperature is linear proportional to the radiance

- In the microwave, surface emissivities are low => need to account for reflection (i.e., the portion of microwave radiation emitted by the atmosphere toward the ocean is reflected back to the atmosphere and can be polarized depending on the viewing direction).

Eq.[14.5] can modified to give the brightness temperature $T_{b,\nu}$ measured by a satellite passive microwave detector at a wavenumber **ν**

$$T_{b,\nu} = \varepsilon_\nu^p T_{sur} \exp(-(\tau^* / \mu) + \int_0^{\tau^*} T_{atm}(\tau') \exp(-\tau' / \mu) d\tau' / \mu$$

$$+ R_\nu^p \exp(-\tau^* / \mu) \int_0^{\tau^*} T_{atm}(\tau') \exp(-(\tau^* - \tau') / \mu) d\tau' / \mu$$

[14.13]

where

T_{sur} is the surface temperature

T_{atm} is the atmospheric temperature

ε_ν^p is the emissivity of the ocean surface with the given polarization state p;

$R_\nu^p = (1 - \varepsilon_\nu^p)$ is reflectivity of the ocean surface with the given polarization state p;

Let's assume that the absorption by water vapor only in the boundary layer

$$\int_0^{\tau^*} T_{atm}(\tau') \exp(-\tau' / \mu) d\tau' / \mu \approx T_{sur} [1 - \exp(-\tau^* / \mu)]$$

Thus we have from Eq.[14.13]

$$T_{b,\nu} = T_{sur}[1 - T_\nu{}^2(\tau^* / \mu)(1 - \varepsilon_\nu^p)] \qquad [14.14]$$

where $T_\nu(\tau^* / \mu) = \exp(-\frac{\tau_\nu^*}{\mu})$

3. Measurements of path-integrated quantities (precipitable water vapor, cloud liquid water).

Let's consider brightness temperature measured at 19.35 GHz and 37 GHz for two measured polarization state (horizontal, H, and vertical V, polarization states)
Using Eq.[14.14], we have at each frequency

$$\Delta T_{b,\tilde{\nu}} = T_{sur}(R^V - R^H) T_{\tilde{\nu}}{}^2(\tau^* / \mu) \qquad [14.15]$$

where

$$R_{\tilde{\nu}}^{H,V} = (1 - \varepsilon_{\tilde{\nu}}^{H,V})$$

The atmospheric transmission can be represented as a combination of transmission for O_2, T_{o2} , cloud liquid water, T_W, and water vapor T_ϖ at each frequency

$$T^2 = T_{o2}^2 T_W^2 T_\varpi^2$$

$T_W = \exp(-k_{a,W} LWP / \mu)$ where $k_{a,W}$ is the absorption coefficient of liquid water (cloud drops) and *LWP* is the liquid water path: liquid water content (*LWC*, Lecture) integrated over the path

$T_\varpi = \exp(-k_\varpi \varpi / \mu)$ where k_ϖ is the absorption coefficient of water vapor and ϖ is the amount of water vapor integrated over the part (called precipitable water)

From Eq.[14.15] we have

$$k_{a,W}LWP + k_{\varpi}\varpi = -\frac{\mu}{2}\ln[\frac{\Delta T_b}{T_{sur}(R^V - R^H)T_{02}^2}] \qquad [14.16]$$

- Eq.[14.16] for two channels=> we have two equations to solve for *LWP* and ϖ given the values of $T_{o2,19}$, $T_{o2,37}$, $k_{a,W}$, k_{ϖ}, and $R_{\tilde{V}}^{H,V}$

Problems:

1) absorption coefficients
2) $R_{\tilde{V}}^{H,V}$ are functions of wind speed

- The above principle is used in the retrieval algorithm of Special Sensor Microwave/Imager (SSM/I)

SSM/I is a passive microwave sensor aboard the DMSP satellite series
http://www.remss.com/

The SSM/I dataset consists of:

F08 SSM/I	Jul 1987	to	Dec 1991
F10 SSM/I	Jan 1991	to	Nov 1997
F11 SSM/I	Jan 1992	to	May 2000
F13 SSM/I	May 1995	to	present
F14 SSM/I	May 1997	to	present
F15 SSM/I	Dec 1999	to	present

Example of SSM/I products