Lecture 18

Principles of sounding by emission.

Objectives:

1. Concept of weighting functions.
2. Weighting functions for nadir sounding.
3. Weighting functions for limb sounding.

Required reading:

G: 7.5

Additional reading:

COMET tutorial

http://cimss.ssec.wisc.edu/goes/comet/17.html

Advanced reading:

C. D. Rodgers, Inverse methods for atmospheric sounding: Theory and practice. 2000.

1. Concept of weighting functions.

Recall Eq.[14.5] which gives the **upward intensity in the plane-parallel atmosphere with emission (neglecting scattering):**

$$I_\nu^{\uparrow}(\tau,\mu) = I_\nu^{\uparrow}(\tau^*,\mu)\exp(-\frac{\tau^*-\tau}{\mu})$$

$$+\frac{1}{\mu}\int_{\tau}^{\tau^*}\exp(-\frac{\tau'-\tau}{\mu})\ B_\nu(\tau')d\tau'$$

Let's introduce the transmittance along the path from the optical depth **τ`** to the optical depth **τ** as

$$T_\nu(\tau',\tau,\mu) = \exp(-\frac{\tau'-\tau}{\mu}) \qquad [18.1]$$

where **μ** is the cosine of solar zenith angle of observation. Thus

$$\frac{dT_\nu(\tau',\tau,\mu)}{d\tau'} = -\frac{1}{\mu}\exp(-\frac{\tau'-\tau}{\mu})$$

Therefore, Eq.[14.5] can be re-written as

$$I_\nu^{\uparrow}(\tau,\mu) = I_\nu(\tau^*)T_\nu(\tau^*,\tau,\mu) + \int_{\tau^*}^{\tau} B_\nu(\tau')\frac{dT_\nu(\tau',\tau,\mu)}{d\tau'}d\tau' \qquad [18.2]$$

Eq.[18.2] can be expressed in different vertical coordinates such as z, p or ***ln***(p). Let's denote an arbitrary vertical coordinate by $\tilde{z}$, so Eq.[18.2] becomes

$$I_\nu^{\uparrow}(\tilde{z},\mu) = I_\nu(\tilde{z}^*)T_\nu(\tilde{z}^*,\tilde{z},\mu) + \int_{\tilde{z}^*}^{\tilde{z}} B_\nu(\tilde{z}')W(\tilde{z}',\tilde{z},\mu)d\tilde{z}' \qquad [18.3]$$

where we introduced the weighting function as

$$\boxed{W_\nu(\tilde{z}_1,\tilde{z}_2,\mu) = \frac{dT_\nu(\tilde{z}_1,\tilde{z}_2,\mu)}{d\tilde{z}}} \qquad [18.4]$$

Physical meaning of the weighting function:

Radiances emitted from a layer $d\tilde{z}'$ is determined by a blackbody emission, $B_\nu(\tilde{z}')$, of the layer weighted by the factor $W_\nu(\tilde{z}',\tilde{z},\mu)d\tilde{z}'$

Let's re-write the solutions of the radiative transfer equation for upward and downward radiances in the altitude coordinate, z. Recall (see Lecture 7) that the optical depth of a layer due to absorption by a gas in this layer is

$$\tau_\nu = \int_{u_1}^{u_2} k_\nu du$$

Let's express the optical depth in terms of a mass absorption coefficient of the absorbing gas and its density

$$\tau_\nu = \int_{z'}^{z} k_\nu \rho_{gas} dz \qquad [18.5]$$

Thus transmission between z and z’ along the path at μ is

$$T_\nu(z, z', \mu) = \exp(-\frac{1}{\mu}\int_{z'}^{z} k_\nu \rho_{gas} dz'') \qquad [18.6]$$

and

$$\frac{dT_\nu(z, z', \mu)}{dz'} = -\frac{k_\nu \rho_{gas}}{\mu}\exp(-\frac{1}{\mu}\int_{z'}^{z} k_\nu \rho_{gas} dz'') \qquad [18.7]$$

Therefore for the **upward intensity and downward intensities** in the altitude z coordinate, we have

$$I_\nu^{\uparrow}(z, \mu) = I_\nu^{\uparrow}(0; \mu,)T_\nu(z,0,\mu) + \int_0^z B_\nu(T(z'))\left|\frac{dT_\nu(z, z', \mu)}{dz'}\right| dz' \qquad [18.8]$$

$$I_\nu^{\downarrow}(z,-\mu) = \int_z^\infty B_\nu(T(z'))\left|\frac{dT_\nu(z, z', \mu)}{dz'}\right| dz' \qquad [18.9]$$

$$I_\nu^{\uparrow}(z, \mu) = I_\nu^{\uparrow}(0; \mu,)\exp\left[-\frac{1}{\mu}\int_0^z k_\nu \rho_{gas} dz\right]$$
$$+\frac{1}{\mu}\int_0^z \exp\left[-\frac{1}{\mu}\int_{z'}^{z} k_\nu \rho_{gas} dz'\right] B_\nu(T(z')) k_\nu \rho_{gas} dz' \qquad [18.10]$$

$$I_\nu^{\downarrow}(z,-\mu) = \frac{1}{\mu}\int_z^\infty \exp\left[-\frac{1}{\mu}\int_{z'}^{z} k_\nu \rho_{gas} dz'\right] B_\nu(T(z')) k_\nu \rho_{gas} dz' \qquad [18.11]$$

2. Weighting functions for near-nadir sounding

From Eq.[18.8], upwelling radiance detected by a satellite sensor at $Z=\infty$ is

$$I_\nu^\uparrow(\infty,\mu) = I_\nu^\uparrow(0;\mu,)T_\nu(\infty,0,\mu) + \int_0^\infty B_\nu(T(z'))\left|\frac{dT_\nu(\infty,z',\mu)}{dz'}\right|dz'$$

or

$$I_\nu^\uparrow(\infty,\mu) = I_\nu^\uparrow(0;\mu,)T_\nu(\infty,0,\mu) + \int_0^\infty B_\nu(T(z))W_\nu(\infty,z,\mu)dz$$

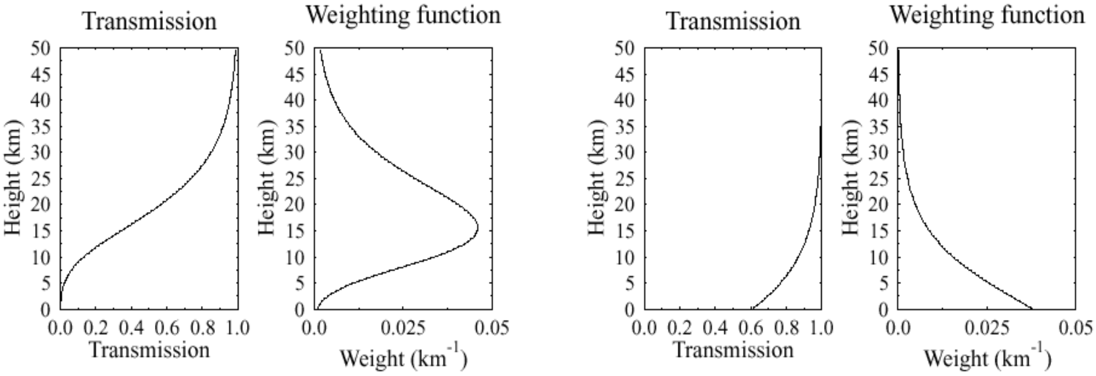
where

$$W_\nu(\infty,z,\mu) = \left|\frac{dT_\nu(\infty,z,\mu)}{dz}\right|$$

thus from Eq.[18.7] we have

$$W_\nu(\infty,z,\mu) = \left|\frac{T_\nu(\infty,z,\mu)}{dz}\right| = \underbrace{\boxed{\frac{k_\nu \rho_{gas}}{\mu}}}_{\text{decreases with altitude}}\underbrace{\boxed{\exp(-\frac{1}{\mu}\int_z^\infty k_\nu \rho_{gas} dz)}}_{\text{increases with altitude}} \qquad [18.12]$$

Schematic of weighting function for optically thick and optically thin media



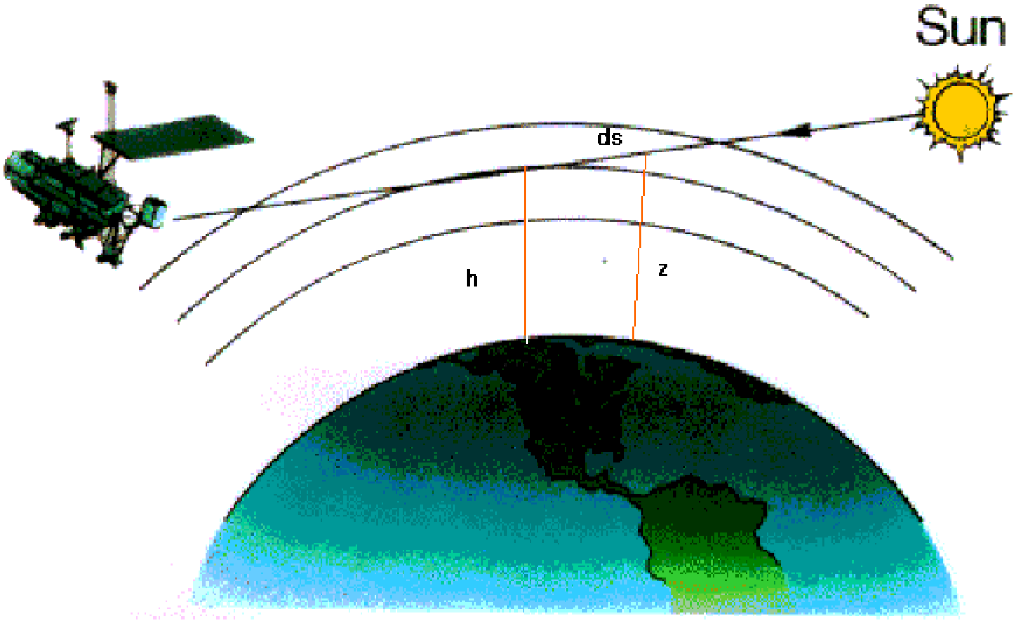
3. Weighting functions for limb sounding

The intensity measured by a satellite for limb viewing geometry is the integral of the emission along a line-of-sign

$$I_{\nu}(h) = \int_{\infty}^{0} B_{\nu}(s) \frac{dT_{\nu}(s,0)}{ds} ds \qquad [18.13]$$

h is the tangent altitude,

T_ν(s, 0) is the transmission function along the path of length s from the outer levels of the atmosphere at $s = \infty$



We can relate ***s*** and ***z*** as

$$(R + h)^2 + s^2 = (R + z)^2$$

where R is the radius of the Earth. Using that R>>h, we have

$$s^2 \approx 2R(z - h)$$

Thus Eq.[18.13] becomes

$$I_{\nu}(h) = \int_{h}^{\infty} B_{\nu}(z') \frac{dT_{\nu}(z', \infty)}{ds} \frac{ds}{dz'} dz'$$

and $$I_\nu(h) = \int_h^\infty B_\nu(z')W(h,z',\infty)dz'$$

where

$$W_\nu\,(h,z,\infty) = 0 \qquad \text{for } \boldsymbol{z} < \boldsymbol{h}$$

$$W_\nu\,(h,z,\infty) = \frac{dT_\nu(z,\infty)}{ds}\boxed{\sqrt{R/2(z-h)}} \qquad \text{for } \boldsymbol{z} > \boldsymbol{h}$$

↑

enhancement of the tangent path relative to a vertical path

⇩

limb sounding has higher sensitivity to emission from trace gases (CO, NO, N_2O, ClO)

Advantages of limb sounding:

- Can measure emission from gases of low concentrations
- Surface emission does not effect limb sounding
- Good vertical distribution since it senses outgoing radiation from just a few kilometers above the tangent height (=> weighting functions have spikes, see G:fig.7.18)

Disadvantages of limb sounding:

- Can not be used below the troposphere
- Requires precise information of the viewing geometry