Lecture 19

Sounding of the temperature profile, trace gases and air pollution.

Objectives:

1. Concept of an inverse problem.
2. Sounding of the atmospheric temperature. NOAA sounders
3. Sounding of atmospheric gases.

Required reading:

G: 1.2, 7.5.4, 7.7

Additional reading:

Kidder and Vonder Haar: Chapter 6

Advanced reading:

Houghton J.T., F.W Taylor, and C. D. Rodgers, Remote sounding of atmospheres. 1984.

1. Concept of an inverse problem.

Recall Eq.[14.5] which gives the solution of the radiative transfer equation with emission for the **outgoing intensity** at the top of the atmosphere $Z=\infty$

$$I_\nu^\uparrow(\infty,\mu) = I_\nu^\uparrow(0;\mu,)T_\nu(\infty,0,\mu) + \int_0^\infty B_\nu(T(z))W_\nu(\infty,z,\mu)dz$$

Forward problem is to calculate outgoing radiances for given temperature and gas concentration profiles.

Inverse problem in remote sounding is to determine what temperature and gas concentration profiles could have produced a set of radiances observed at closely spaced wavenumbers.

Let's assume that $T_\nu(\infty,0,\mu)$ is negligible small. We have for nadir sounding

$$I_{\nu_i}^{\uparrow}(\infty) = \int_0^{\infty} B_{\nu_i}(T(z))W_{\nu_i}(\infty, z)dz \qquad [19.1]$$

where ν_i , i =1, 2…N, is the wavenumbers (i.e., centers of the finite spectral bands of a spectrometer)

We can ignore the frequency dependence of the Planck function if ν_i are closely spaced. So we can re-write Eq.[19.1] as

$$I_{\nu_i}^{\uparrow}(\infty) = \int_0^{\infty} B(\overline{\nu}, T(z))W_{\nu_i}(\infty, z)dz \qquad [19.2]$$

Consider that measurements are done in the CO2 absorbing band, so the weighting function are known and $B(\overline{\nu}, T(z))$ is unknown. Thus we want to solve Eq.[19.2] to find $B(\overline{\nu}, T(z))$ and hence $T(z)$.

Eq.[19.2] is a Fredholm integral equation of the first kind, which has long been known for many associated difficulties. The general form of the Fredholm integral equation (because the limits of the integral are fixed , not variable) of the first kind (because ***f(x)*** appears only in the integral) is

$$g(y) = \int_a^b f(x)K(y, x)dx \qquad [19.3]$$

where

f(x) is unknown (in our case $B(\overline{\nu}, T(z))$

K(y, x) is called the kernel (or kernel function), (in our case $W_{\nu_i}(\infty, z)$)

g(x) is known (in our case $I_{\nu_i}^{\uparrow}(\infty)$)

Problems in solving the inverse problem (i.e., problems in solving Eq.[19.2])

- The inverse problem is ill-posed (i.e., underconstrained) because there are only a finite number of measurements and the unknown is a continuous function.
- This inverse problem is ill-conditioned (i.e., any experimental error in the measurements of radiances can be greatly amplified, so that the solution becomes meaningless)

2. Sounding of the atmospheric temperature.

Let's find the simplest solution of Eq.[19.2], assuming that transmittance (and hence weighting functions) does not depend on temperature. Thus Eq.[19.2] is linear in $B(\overline{\nu}, T(z))$.

A standard approach is to express $B(\overline{\nu}, T(z))$ as a linear function of ***N*** variables, b_j:

$$B(\overline{\nu}, T(z)) = \sum_{j=1}^{N} b_j L_j(z) \qquad [19.4]$$

where $L_j(z)$ is a set of representation functions such as polynomials or sines and cosines.

Eq.[19.2] becomes

$$I^{\uparrow}_{\nu_i} = \sum_{j=1}^{N} b_j \int_0^{\infty} L_j(z) W_{\nu_i}(z) dz = \sum_{j=1}^{N} C_{ij} b_j \qquad [19.5]$$

where the square matrix C whose elements $C_{ij} = \int_0^{\infty} L_j(z) W_{\nu_i}(z) dz$ can be easily calculated. Thus for the N unknown b_j there are N equations which can be solved. The solution of Eq.[19.5] is called an **exact solution to the linear problem.**

Problems:

Eq.[19.5] is ill-conditioned.

Strategy: instead of **an exact solution**, find the solution that lies within the experimental error of the measurements => it gives us more freedom in a choice of a solution, but a new problem is how to make this choice

Thus the problem of retrieval can be re-stated as:

Given the measured radiances, the statistical experimental error, the weighting functions, and any other relative information, what solutions for $B(\bar{\nu}, T(z))$ are physically meaningful?

A priori information is often used to provide "other relative information".

A priori information for the retrievals of temperature:

- Radiosonde data
- Forecast atmospheric dynamical models

Examples: GOES soundings are generated every hour, using an ETA forecast as a 'first guess'

<u>One of many linear methods:</u>

Minimum variance method is used for routine soundings from the TOVS (TIROS N Operational Vertical Sounder):

Strategy: to seek the solution which minimizes the mean-square differences between the retrieved profile and true profiles

The radiative transfer equation is linearized about a true profile, so a matrix equation can be constructed

$$\mathbf{l} = \mathbf{At} + \mathbf{e}$$

where **l** is the column vector of radiance deviations from the true profile,

t is the vector of temperature deviations from the true profile;

e is the column vector of measurement errors;

A is the matrix containing the weighting functions, the Planck sensitivity factors dB/dT and it can be calculated with a knowledge of the transmittances.

Let's assume at the moment that we have a set of collocated radiosonde and satellite observations

$$\mathbf{L} = \mathbf{AT} + \mathbf{E} \qquad [19.6]$$

here upper cases means that matrices are N columns for N sounding pairs.

We seek a matrix C that **T = CL.** It can be shown that the matrix C which, in a least-squares sense, minimizes errors in **T = CL** is

$$\mathbf{C = TL^T (LL^T)^{-1}} \qquad [19.7]$$

where the superscript T indicates matrix transpose and (-1) indicates the matrix inverse. Substituting Eq. [19.6] in Eq.[19.7], we have

$$\mathbf{C = T(AT + E)^T [(AT+E)(AT+ E)^T)]^{-1}} \qquad [19.8]$$

Expanding and using that $(AB)^T=B^TA^T$, it becomes

$$\mathbf{C = T(T^TA^T + E^T) [AT\, T^TA^T +ATE^T + ET^TA^T + EE^T]^{-1}} \qquad [19.9]$$

Assuming that the measurement errors are uncorrelated with temperature deviations, $\mathbf{TE^T}$ and $\mathbf{ET^T}$ are negligible small and therefore

$$\mathbf{C = S_TA^T[AS_TA^T +S_E]^{-1}} \qquad [19.10]$$

$\mathbf{S_T}$ the temperature covariance matrix

$\mathbf{S_E}$ the radiance error covariance matrix

Eq.[19.10] is called minimum variance method.

NOAA sounders:

NOAA-series polar orbiting satellites:

HIRS/2 (High Resolution Infrared Radiation Sounder 2): 20 channels, resolution 19 km (nadir)

MSU (Microwave sounding Unit): 4 channels, resolution 111 km (nadir)

SSU (Stratospheric Sounding Unit): 3 channels (in the 15-μm CO_2 band), resolution 111 km (nadir)

[HIRS+MSU+SSU] is called TOVS (TIROS N Operational Vertical Sounder)

NOTE: MSU+ SSU were replaced by AMSU-A and AMSU-B (Advanced Microwave Sounding Units) on NOAA K, L, M series

3. Sounding of atmospheric gases.

Strategy: make satellite measurements at frequencies in absorption bands of atmospheric gases

The principles of the methods for gas retrievals are generally the same as for temperature retrievals, but gas inversion is more difficult:

- The inverse problem is more nonlinear for constituents (because they enter the radiative transfer equation through the mixing ratio profile in the exponent. It is not possible to separate the radiative transfer equation into the product of a simple constituent function and one which is constituent-independent).
- The second main problem is that, in some cases, the radiances are insensitive to changes in the mixing ratio. (For instance, for an isothermal atmosphere at temperature T, any mixing ratio profile will result in the same radiances (i.e. $B_\nu(T)$). In practice, we find this problem in the retrieval of low-level water vapor; because the temperature of the water vapor is close to that of the surface, infrared radiances are relatively insensitive to changes in low-level water vapor).

Current satellite sensors/missions:

GOME (Global Ozone Monitoring Experiment), ESA-ERS, 1995-present
retrievals of O3, NO2, H2O, BrO, SO2, HCHO, OCLO

SCIAMATCHY (Scanning Imaging Absorption Spectrometer for Atmospheric Cartography), ESA-ENVISAT, 2001-present
retrievals of O2, O3, NO, NO2, N2O, H2O, BrO, SO2, HCHO, H2CO, CO, OCLO, CO2, CH4

MOPITT
http://www.eos.ucar.edu/mopitt/
retrievals of CO and CH4

New NASA satellite missions:
NASA Aura
http://eos-aura.gsfc.nasa.gov/news/index.html

HIRDLS is an infrared limb-scanning radiometer designed to sound the upper troposphere, stratosphere, and mesosphere to determine temperature; the concentrations of O3, H2O, CH4, N2O, NO2, HNO3, N2O5, ClONO2, CFCl2, CFCl3, and aerosols; and the locations of polar stratospheric clouds and cloud tops.