Lecture 4

Blackbody radiation. Main Laws. Brightness temperature.

Objectives:

1. Concepts of a blackbody, thermodynamical equilibrium, and local thermodynamical equilibrium.
2. Main laws:
   - Blackbody emission: Planck function.
   - Stefan-Boltzmann law.
   - Wien's displacement law.
   - Kirchhoff's law.
3. Brightness temperature.
4. Sun as an energy source. Solar constant.

Required reading:

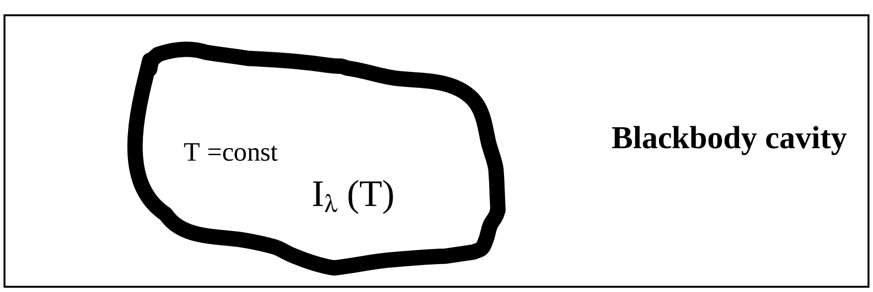
G 2.5

1. Concepts of a blackbody and thermodynamical equilibrium.

Blackbody is a body whose surface absorbs all radiation incident upon it.

Thermodynamical equilibrium describes the state of matter and radiation inside an isolated constant-temperature enclosure.

Blackbody radiation is the radiative field inside a cavity in thermodynamic equilibrium.



NOTE: A blackbody cavity is an important element in the design of radiometers. Cavities are used to provide a well-defined source for calibration of radiometers. Another use of a cavity is to measure the radiation that flows into the cavity (e.g., to measure the radiation of sun).

***Properties of blackbody radiation*:**

- Radiation emitted by a blackbody is isotropic, homogeneous and unpolarized;
- Blackbody radiation at a given wavelength depends only on the temperature T;
- Any two blackbodies at the same temperature emit precisely the same radiation;
- A blackbody emits more radiation than any other type of an object at the same temperature;

NOTE: The atmosphere is not strictly in the thermodynamic equilibrium because its temperature and pressure are functions of position. Therefore, it is usually subdivided into small subsystems each of which is effectively isothermal and isobaric referred to as **Local Thermodynamical Equilibrium (LTE).**

- A concept of **LTE** plays a fundamental role in atmospheric studies: e.g., the main radiation laws discussed below, which are strictly speaking valid in **thermodynamical equilibrium**, can be applied to an atmospheric air parcel in **LTE.**

2. Main laws.

Blackbody Emission

Planck function, $B_\lambda(T)$, gives the **intensity (or radiance)** emitted by a blackbody having a given temperature.

- **Plank function** can be expressed in wavelength, frequency, or wavenumber domains as

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5(\exp(hc/k_B T\lambda) - 1)} \quad [4.1]$$

$$B_{\tilde{\nu}}(T) = \frac{2h\tilde{\nu}^3}{c^2(\exp(h\tilde{\nu}/k_B T) - 1)} \quad [4.2]$$

$$B_\nu(T) = \frac{2h\nu^3 c^2}{\exp(h\nu c/k_B T) - 1} \quad [4.3]$$

where λ is the wavelength; $\tilde{\nu}$ is the frequency; ν is the wavenumber;
$\boldsymbol{h}$ is the Plank's constant; $\boldsymbol{k_B}$ is the Boltzmann's constant (k_B = 1.38x10^{-23} J K^{-1});
$\boldsymbol{c}$ is the velocity of light; and $\boldsymbol{T}$ is the absolute temperature of a blackbody.

NOTE: The relations between $B_{\tilde{\nu}}(T)$; $B_\nu(T)$ and $B_\lambda(T)$ are derived using that

$$I_{\tilde{\nu}} d\tilde{\nu} = I_\nu d\nu = I_\lambda d\lambda, \text{ and that } \lambda = c/\tilde{\nu} = 1/\nu$$

$$\Downarrow$$

$$B_{\tilde{\nu}}(T) = \frac{\lambda^2}{c} B_\lambda(T)$$

$$B_\nu(T) = \lambda^2 B_\lambda(T)$$

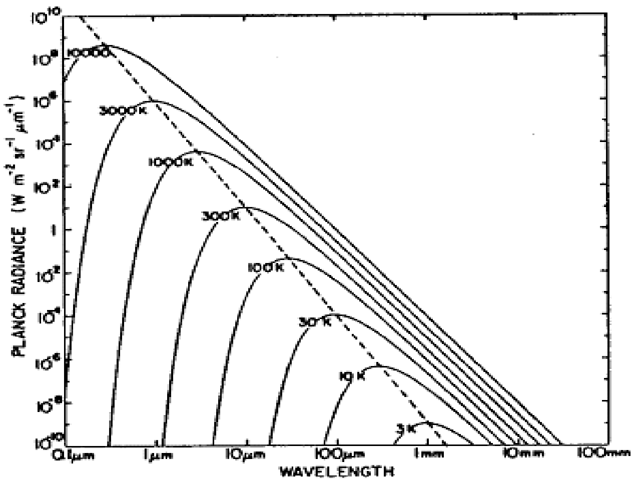


Figure 4.1
Planck radiances versus wavelengths for the

Asymptotic behavior of Planck function:

- **If λ $\rightarrow \infty$ (or $\tilde{\nu}$ -> 0) (known as Rayleigh –Jeans distributions):**

$$B_\lambda(T) = \frac{2k_B c}{\lambda^4} T \qquad [4.4a]$$

$$B_{\tilde{\nu}}(T) = \frac{2k_B \tilde{\nu}^2}{c^2} T \qquad [4.4b]$$

NOTE:

- ✓ **Rayleigh –Jeans distributions** has a direct application to passive **microwave** remote sensing.
- ✓ For large wavelengths, the emission is directly proportional to T .

- **If λ -> 0 (or $\tilde{\nu}$ $\rightarrow \infty$):**

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \exp(-hc/\lambda k_B T) \qquad [4.5a]$$

$$B_{\tilde{\nu}} = \frac{2h\tilde{\nu}^3}{c^2} \exp(-h\tilde{\nu}/k_B T) \qquad [4.5b]$$

Stefan-Boltzmann law.

The **Stefan-Boltzmann law** states that the total power (energy per unit time) emitted by a **blackbody,** per unit surface area of the **blackbody**, varies as the fourth power of the temperature.

$$\mathbf{F = \pi B(T) = \sigma_b T^4}$$

where σ_b is the *Stefan-Boltzmann constant* (σ_b = 5.671x10^{-8} W m^{-2} K^{-4}),

F is energy flux [W m^{-2}], and **T** is blackbody temperature [K];

and **B(T) =** $\int_0^{\infty} B_{\lambda}(T)\, d\lambda$

Wien's displacement law.

The **Wien's displacement law** states that the wavelength at which the blackbody emission spectrum is most intense varies inversely with the blackbody's temperature. The constant of proportionality is Wien's constant (2897 K μm):

$$\lambda_m = 2897 / T$$

where λ_m is the wavelength (in micrometers, μm) at which the peak emission intensity occurs, and **T** is the temperature of the blackbody (in degrees Kelvin, K).

NOTE: This law is simply derived from $dB_{\lambda}/d\lambda = 0$

NOTE: Easy to remember statement of the Wien's displacement law:
the hotter the object the shorter the wavelengths of the maximum intensity emitted

Kirchhoff's law.

The **Kirchhoff's law** states that the emissivity, ε_λ, of a medium is equal to the absorptivity, A_λ, of this medium under thermodynamic equilibrium:

$$\varepsilon_\lambda = A_\lambda$$

where ε_λ is defined as the ratio of the emitting intensity to the Planck function;

A_λ is defined as the ratio of the absorbed intensity to the Planck function.

For a **blackbody**: $\boxed{\varepsilon_\lambda = A_\lambda = 1}$

For a **gray body:** $\varepsilon = A < 1$ (i.e., no dependency on the wavelength)

For a **non-blackbody**: $\varepsilon_\lambda = A_\lambda < 1$

NOTE: **Kirchhoff's law** applies to gases, liquids and solids if they in TE or LTE.

- In remote sensing applications, one needs to distinguish between the **emissivity of the surface** (e.g., various types of lands, ice, ocean etc., see Lecture 5) and the **emissivity of an atmospheric volume** (consisting of gases, aerosols, and/or clouds, see Lecture 7).

3. Brightness temperature is the key concept in remote sensing.

Brightness temperature, T_b, is defined as the temperature of a blackbody that emits the same intensity as measured. Brightness temperature is found by inverting the Planck function. For instance, from Eq.[4.1]:

$$T_b = \frac{C_2}{\lambda \ln[1 + \frac{C_1}{\lambda^5 I_\lambda}]}$$

where $C_1 = 1.1911 \times 10^8$ W m^{-2} sr^{-1} μm^4

$C_2 = 1.4388 \times 10^4$ K μm

- For a blackbody: brightness temperature = kinetic temperature ($T_b = T$)
- For natural materials: $T_b^4 = \varepsilon T^4$ (ε is the broadband emissivity)

NOTE: In the microwave region, the Rayleigh –Jeans distributions gives $T_b = \varepsilon_\lambda T$.

However, ε_λ is a complex function of several parameters (see Lecture 5)

4. Sun as an energy source. Solar constant.

- Solar flux reaching the earth is a function of time determined by

  1) the orbital characteristics of the earth and the sun (i.e., eccentricity; obliquity, and periodic precession)

  2) the sun properties (e.g., solar surface activity).

NOTE:

a) Sun is a gaseous sphere consisting of hydrogen, helium, iron, silicon, etc.

Solar energy: nuclear fusion (conversion of four hydrogen atoms to one helium atom)

b) Temperature of sun's photosphere is about 5800 K.

b) Sunspots are cooler regions of the sun (with T = 4000K). Period between sunspot maxima is about 11 years (called **11-year-cycle**).

Solar constant, S_0, is defined as total flux of solar energy, reaching the top of the atmosphere, per unit surface normal to the solar beam at the mean distance between the sun and the earth.

The sun emits about $F_{sun} = 6.2\text{x}10^7$ W/m^2. On the basis of energy conservation law, we have

$$F_{sun}\, 4\,\pi\, r_{sun}^{\ 2} = S_0\, 4\,\pi\, d_0^{\ 2}$$

where r_{sun} is the radius of the sun ($6.96\text{x}10^5$ km), and d_0 is the mean distance between the sun and the earth ($1.5\text{x}10^8$ km). Hence,

$$\boxed{S_0 = F_{sun}\, (r_{sun}/d_0)^2}$$

Mean measured value **S_0 = 1366** W m^{-2} with the measurement uncertainty +/ 3W m^{-2}.

Actual solar flux at the top of the atmosphere at a given time is

$$F_o = S_0 \left(\frac{d_o}{d} \right)^2 \cos(\theta_o)$$

where $\mathbf{d_o}$ is the mean distance from the center of the sun to the earth and

d is the actual distance on a given day (depends of the earth orbit).

Solar zenith angle cos (θ_0):

$$\cos\theta_0 = \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(h)$$

where ϕ is the latitude;

δ is the solar inclination angle (varies throughout the seasons)

h is the local hour angle of the sun (***h=0*** at local solar noon).